1Capturing Heterogeneity in Mode Choice Decisions: Comparing and Combining Machine **2Learning and Discrete Choice Models** 3 4 5 6 **8Panagiotis Tsoleridis** 9Institute for Transport Studies 10University of Leeds, Leeds, United Kingdom, LS2 9JT 11Email: ts17pt@leeds.ac.uk 12 13Charisma F. Choudhury **14**Institute for Transport Studies 15University of Leeds, Leeds, United Kingdom, LS2 9JT 16Email: C.F.Choudhury@leeds.ac.uk 17 **18Stephane Hess** 19Institute for Transport Studies 20University of Leeds, Leeds, United Kingdom, LS2 9JT 21Email: S.Hess@leeds.ac.uk 22 23 24 25 26 27 28 29

1ABSTRACT

In the era of big data, machine learning (ML) has emerged as a strong competitor of econometric 3modelling, discrete choice models (DCMs) in particular. However, a key limitation of the purely data 4driven models is the lack of valuation measures – for example the difficulty in calculation of value of 5time to feed into the cost-benefit analyses. The current study focuses on combining ML-based 6segmentation approaches with DCM and tests their performances against latent class choice models 7(LCCM). LCCM allows the simultaneous allocation of individuals to a predefined number of classes 8alongside the estimation of their mode choice behavior. For the combined ML and DCM, this is done as a 9two-stage approach, where individuals are first allocated into clusters using a deterministic K-modes/K-10means algorithm and then a mode choice model is estimated per cluster. In both cases, the underlying 11mode choice component is specified as a multinomial logit model. The dataset utilized in the current 12study includes the trips of 540 unique individuals, 12524 in total, captured through GPS traces collected 13using a smartphone app. The results suggest that LCCM performs significantly better than both variants 14of the ML counterpart in terms of model fit. It also has clearer insights about the segment compositions. 15The latter, however, results in a much faster estimation and produces reasonable Value of Time estimates 16and demand elasticities. The findings are expected to provide guidance to researchers and practitioners in 17choosing the most efficient method to capture the taste heterogeneity among the segments of the travelers.

19**Keywords:** Unsupervised learning, Clustering, K-modes algorithm, Latent classes, mode choice 20modelling

1 1. INTRODUCTION

During the last decade the abundance of data has provided not only the potential for further 3research advancements, but also posed challenges as to how to derive value out of those emerging data 4sources. In the field of transport, specifically, different forms of data, such as GPS traces, Call Detail 5Records, geotagged tweets etc. require significant pre-processing and knowledge that transcends into 6different fields of research (e.g. computer science) (Antoniou et al., 2019). The massive size of the data 7has also led to increase in the popularity of Machine Learning (ML) techniques over traditional 8econometric models (Discrete Choice Modelling (DCM) techniques for instance). This has led to the 9necessity of comparing and contrasting ML with traditional DCM.

10 Originating from the field of Computer Science, ML algorithms are generally characterised as 11non-parametric methods (with some exceptions) aiming to minimise the errors between actual and 12predicted outcomes without relying on any behavioural assumptions of the underlying model. ML 13encompasses a large array of algorithms, which can be broadly categorised into supervised and 14unsupervised learning. The majority of comparative studies in the literature between ML and DCM focus 15on comparing supervised ML algorithms, e.g. Artificial Neural Networks and Random Forests, with 16DCM frameworks, such as MNL and Nested Logit, usually in the context of mode choice (Hensher and 17Ton, 2000; Xie et al., 2003; Cantarella and de Luca, 2005; Zhang and Xie, 2008; Sekhar et al., 2016; 18Hagenauer and Helbich, 2017). Their findings suggest that ML algorithms have the potential to be used as 19an alternative method for behavioural modelling due to their superior predictive performance, although 20Hensher and Ton (2000) also highlight the limitations associated with the lack of interpretable results 21compared to a DCM framework. Furthermore, a wide range of studies has implemented clustering 22algorithms to analyse individual behaviour and uncover mobility patterns (see Anda et al. 2017 for 23details). Though such studies provide good insights about the state of the network, they have limited 24applications in the context of predictions and/or valuation (e.g. calculation of value of time to feed into 25the cost-benefit analyses).

A key advantage of an ML-based approach is the efficiency in capturing the patterns in the data 27which can then be used in segmenting the travelers and capturing the taste heterogeneity in the sample. 28Several studies have used clustering techniques for market/sample segmentation (Salomon and Ben-29Akiva, 1983; Lanzendorf, 2002; Krizek and Waddell, 2003) and reported that different lifestyle clusters 30(identified empirically) have different choice elasticities. However, these studies have three key 31limitations. *Firstly*, the previous studies relied on "traditional" samples with regard to their data 32collection methods (e.g. single RP choice scenarios, short trip diaries, etc.) and it is worth investigating 33the performance of similar approaches with passively collected larger samples (more participants and/or 34longer panels). *Secondly*, these studies did not compare the model performances with the more advanced 35discrete choice models that account for heterogeneity among groups of decision makers - Latent Class 36Choice Modelling (LCCM) for example. *Thirdly*, the studies compared the goodness of fit and/or 37prediction capabilities of ML and DCM as opposed to in-depth efforts to formulate models that has the 38best of both worlds – computational advantages on ML and the behavioral interpretation of DCM that can 39be used for valuation.

The current research aims to address this research gap by comparing the performance of different 41sample segmentation techniques with a discrete choice model as the base for the actual choice 42component. The aim is to find the best method to capture the heterogeneity among the travelers and 43simultaneously have outputs that can be used for valuation. The following variants of LCCM and hybrid 44models (ML combined with DCM) have been tested in this regard:

1Model 1: LCCM where the probabilistic sample segmentation (class membership) and mode choices are 2modelled simultaneously.

3Model 2: ML based segmentation using socio-demographic data only combined with DCM for the mode 4choice component

5Model 3: ML based segmentation using socio-demographic and choice data combined with DCM for the 6mode choice component

For all models, the *a priori* assumption is that there is a discrete number of segments 8(classes/clusters); the taste heterogeneity is constant within the same segment but varies among different 9segments. While Model 1 provides a behavioural model specification, where both the individual 10sociodemographic characteristics and the observed mobility choices are taken into consideration, Models 112 and 3 are expected to offer much faster and potentially more efficient segmentation.

The dataset used in the current study is a combination of sociodemographic features and a trip 13diary captured through GPS tracking using a smartphone app (56693 trips from 721 unique individuals in 14the raw dataset). Therefore, the dataset provides a rich level of semantic information in addition to 15capturing a wider spectrum of trips and activities from a set of individuals. Though such passive data 16collection methods have the potential to lead to more accurate estimated parameters of travel behaviour, 17there are challenges associated with deriving the full set of variables that are important from a 18behavioural modelling perspective (e.g. accurate travel times, costs etc.) and defining the choice sets, 19specifically determining the availabilities for the unchosen alternatives. In addition to the contribution 20regarding the comparison of ML- and DCM-based segmentation, the current study also demonstrates how 21these data limitations can be overcome.

The remainder of this paper is structured as follows. In Sections 2 and 3, the methodological 23framework and the data used for the study's practical implementation are described, respectively. Section 244 focuses on the results with focus on the comparison between the different approaches. The main 25conclusions and the direction of future research are summarized in the concluding section.

26

27 **2. METHODOLOGY**

The methodological framework developed for this study involves the implementation of 29probabilistic and deterministic sample segmentation with the use of traditional Discrete Choice Modelling 30(DCM) techniques and Machine Learning (ML) algorithms, respectively. For the former (Model 1), 31Latent Class Choice Modelling (LCCM) is used to simultaneously allocate individuals probabilistically 32into a discrete number of classes, based on selected sociodemographic features, and estimate a choice 33model regarding their mode choice behaviour based on level of service attributes. For the latter, a two-34stage approach is performed by first segmenting the sample deterministically using K-modes/K-means 35clustering and then applying a DCM framework to each cluster separately. The inputs in the K-modes 36clustering vary between Models 2 and 3 with the clustering using only the sociodemographic data and the 37combination of sociodemographic and choice data, respectively.

38 39

2.1 Latent Class Choice Modelling (Model 1)

Discrete Choice Modelling (DCM) has been used extensively in the field of transport, since the 41seminal study of McFadden (1973), inspired by the previous work of Luce (1959) and Marschak (1960) 42linking behavioural choice theory with microeconomics. As described in McFadden (2000), the main 43concept of Random Utility Theory and its basic Multinomial Logit model (MNL) had been extended 44significantly in the following decades leading to the formulation of more behaviourally accurate model 45representations of individual behaviour.

DCM is based on the underlying theory of Random Utility Maximisation (RUM) suggesting that a 2rational individual n is more likely to choose among a set of possible alternatives J the one resulting in 3the maximisation of utility U. The utility $U_{n,j}$ is comprised by a deterministic and a stochastic part. An 4additive form of the Utility function is shown in the following equation (**Equation 1**) (Train, 2009):

5

$$6U_{n,j} = V_{n,j} + \varepsilon_{n,j} \,\forall \, j \in J \tag{1}$$

7

8where $V_{n,j}$ and $\varepsilon_{n,j}$ are the deterministic and stochastic utilities, respectively, for individual n and 9alternative j. The deterministic or systematic part of the utility $V_{n,j}$ is usually formed as a linear-in-the-10parameters function (**Equation 2**) of a vector $X_{n,j}$ containing the observed alternative's j attributes and 11the individual's n characteristics, and their respective taste coefficients $\beta_{n,j}$ including an inherent 12preference towards alternative j as $\beta_{0,j}$, also known as the Alternative Specific Constant (ASC). The 13stochastic part or the error term $\varepsilon_{n,j}$ is considered to include all the uncaptured features that could 14influence the utility of alternative j for individual n, but are currently unknown to the researcher

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$$16V_{n,j} = \beta_{0,j} + \beta_{n,j} X_{n,j} \tag{2}$$

17

LCCM is an extension of DCM which uses an endogenous sample segmentation technique to 19capture the heterogeneity in the sample (Kamakura and Russell, 1989). The basic premise of LCCM is 20that the individuals in the sample can be allocated into a discrete number of classes based on their 21sociodemographic features and their observed behaviour. The LCCM framework includes the estimation 22of a class allocation model, where individuals are allocated into a pre-specified number of classes, and a 23choice model aiming to explain the behaviour of individuals given their class (Kamakura and Russell, 241989; Bhat, 1997; Hess, 2014). Under that framework, the likelihood of individual n is calculated as:

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$$26L_n = \sum_{s=1}^{S} \pi_{n,s} \dot{c} \dot{c} \tag{3}$$

27

28where $\pi_{n,s}$ is the probability that individual n belongs to class s from a total number of S classes and $29P_{j_{n,t}}(\beta_s)$ is the probability that individual n chooses alternative j in choice task t given that he/she belongs 30to class s. The underlying choice model is usually specified as an MNL model, but extensions to more 31advanced modelling specifications are also possible. Under that framework, the taste coefficients β_s are 32the same for each individual in a specific class.

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34 2.2 ML-based sample segmentation techniques (Models 2 & 3)

35 ML unsupervised learning contains a range of algorithms capable of effectively segmenting the 36sample into a discrete number of clusters based on patterns in the dataset. Clustering algorithms can be 37categorised into:

- Centroid-based, such as the "hard" and "soft" K-means/modes algorithms
- Connectivity-based, such as Hierarchical clustering

- Density-based, such as the DBSCAN and OPTICS algorithms
- Distribution-based, such as the Gaussian Mixture Models

- K-means (MacQueen, 1967) and K-modes (Aranganayagi and Thangavel, 2009) are two of the 5most widely-known clustering algorithms that can be applied to continuous and discrete data, 6respectively. As most clustering algorithms, they are based on a distance or (dis-)similarity measure 7between the data points in a dataset. Points with high similarity between them are allocated in the same 8cluster. The points within the same cluster should also be distinguishable from points in other clusters. 9Traditional K-means and K-modes perform a form of "hard" clustering as each point is assigned to a 10unique cluster. Contrary to that the more generalized "soft" or "fuzzy" K-modes algorithm (Kim et al., 112004) assigns a weight or probability to each point and for each cluster. Therefore, each point has a non-12zero probability to belong to any cluster, similar to LCCM. In the current study, "hard" K-means and K-13modes algorithms are implemented and compared with LCCM.
- As a starting point, the K-means/K-modes algorithms choose a predetermined number of random 15points in the multidimensional data space as the initial cluster centroids and the similarities from all data 16points to those centroids are calculated. The points are allocated to their closest centroid and then a new 17cluster centroid is calculated as the mean/mode of all the data points in the cluster. At the next step, new 18similarities are calculated again from all the data points to all the centroids and the algorithm continuous 19iteratively until no more changes occur in the cluster centroids. The underlying purpose of the algorithm 20is to minimise the within-cluster-sum-of-differences (WCSD) and at the same time maximise the 21between-cluster-sum-of- differences (BCSD). That means that points within the same cluster would have 22the greatest degree of similarity, while also being as dissimilar as possible from points in different 23clusters.

24

25 **3. DATA**

The data used for the practical implementation of the current study was collected as part of the 27research project DECISIONS carried out by the Choice Modelling Centre at ITS Leeds, between October 282016-March 2017. Several submodules are included in the dataset as part of the survey capturing various 29aspects of the participants' mobility behaviour, in-home and out-of-home activities, their daily energy 30usage and their social network using a name generator. In addition, the project included a household 31survey module capturing the participants' most important socio-demographic information. The dataset 32and its submodules is thoroughly detailed in Calastri et al. (2018a). In regard to mobility behaviour, the 33individuals' daily trips were captured through GPS tracking using a smartphone app, while the 34participants also had to tag their completed trips with further information regarding their trip purpose and 35mode of transport. As a result, the "DECISIONS" dataset provides a combination of emerging and 36traditional data sources.

37

38 3.1 Data cleaning

Two separate datasets are used for the "DECISIONS" project, one corresponding to the daily trip 40diary captured through GPS tracking and the other for the rest of the project's submodules including the 41socio-demographic component. The former was acquired in the form of an SQL-database file, while the 42latter as an excel sheet. Initially, 56693 trips were included in the raw database file performed by 721 43unique individuals. These trips had to be combined with the sociodemographic file using a unique trip 44identifier number. The pre-processing analysis involved the identification of trip errors in the GPS file.

1More specifically, the trips' timestamps were checked to detect trips starting at the same time or before 2the end of the previous trip resulting in zero or negative activity durations.

At the next stage of data cleaning, trip purposes were examined. The trip purposes, reported by 4the participants, among others included the purpose of "*Change of travel mode*". It was decided to merge 5those journeys together with the following trips, since the subsequent mode choice analysis would focus 6solely on the main mode and not on the access mode. Furthermore, trips with no additional information 7on mode and purpose (untagged trips) were removed and the geographical scope was limited to the region 8of Yorkshire. This led to a cleaned dataset with 38624 observations from 721 respondents. The mode 9alternatives selected for the mode choice analysis were car, bus, rail, taxi, cycling and walking.

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11

12 3.2 Data enrichment

Due to the nature of the data at hand, certain important variables from a behavioural modelling 14perspective were missing (travel cost and travel time for non-chosen alternatives). For that reason, a data 15enrichment process was performed with different components summarised in the following paragraphs.

16 3.2.1 Travel time/distance estimation

17 In the current study, real network travel times and distances for each origin-destination pair and 18for each mode were derived using the Google "Directions" 19(https://developers.google.com/maps/documentation/directions/intro). The API was implemented to 20estimate the travel times/distances both for the chosen and the non-chosen alternatives, since using the 21stated time/distance for the chosen mode and the network travel times/distances for the non-chosen ones 22would have the risk of producing biased estimates (Calastri et al., 2018b). The API provided travel times 23based on the stated time of day, hence the state of the network was taken into consideration. Furthermore, 24travel times per link segment were obtained, instead of a total travel time for the whole trip, with the 25purpose of more accurately estimating fuel consumption.

26 3.2.2 Cost estimation

- For car trips, travel cost was segmented into fuel/operating costs and parking costs. The first two 28were calculated using WebTAG's specifications (Department for Transport, 2014). Taking advantage of 29the per segment travel times and distances derived from the API, car travel cost was initially calculated 30for each trip segment and further aggregated per trip. Location-specific parking costs were defined using 31average parking prices per hour for specific places in the region of Yorkshire, such as CBDs, rail stations 32and the airport. For all other locations, it was assumed that there was no parking cost involved.
- For taxi, the cost was calculated as a combination of the initial charge (fixed price), the average 34kilometre cost (pounds per kilometre) and the average time cost (pounds per minute) taking into account 35the different tariffs during day, night and weekend. Relevant information on taxi costs for Yorkshire was 36found only for the cities of Leeds and Sheffield. Separate taxi travel costs (distance and time costs) were 37calculated using the average prices for Leeds and Sheffield for each trip. The final taxi travel cost was 38calculated as the average of the distance and time cost per trip.
- For bus and rail, two different cost calculations were performed. The first one took into account 40the discounted trip cost for season ticket holders, while for the remaining individuals a fixed fare price 41was applied. For the second calculation, only a fixed fare was applied to each participant regardless of 42their possession of a season ticket. The results of both cost calculations were used at the initial model 43specification stage (MNL estimation) and the respective model results were compared with values 44suggested by WEBTag (Value of Time, bus fare elasticity) for validation purposes. The first approach

1(separate costs for season and non-season ticket holders) was the one that yielded results closer to 2WEBTag's values and was selected for the subsequent more advanced model specifications.

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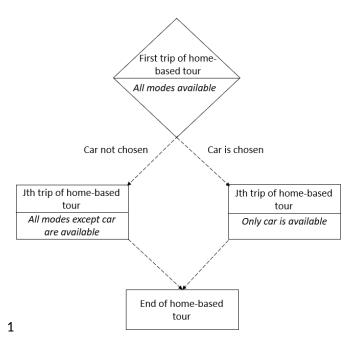
4 *3.2.3 Choice set generation*

In the current study a tour-based approach is followed in contrast to a trip-based one, in which a 6tour is defined as a sequence of trips starting and finishing at the same location (e.g. the individual's 7home location) (as used by Miller et al., 2005; Hasnine and Habib, 2019). Although the modelling 8framework in the current study does not take into consideration the dynamics inside a tour (activity 9schedule, next location etc.), the implemented tour-based approach helps to specify more behaviourally 10accurate mode availability assumptions. Whether or not a mode is included in the choice set hence 11depends on the general availability of the mode (person-specific), consideration of the mode (trip-12specific) (Calastri et al., 2018b) and feasibility of using the mode (restrictions imposed by earlier choices 13made in the same tour).

For person-specific mode availability, the participants' stated availability for car and cycling was 15taken into consideration, while the remaining alternatives (taxi, bus, rail and walking) were considered to 16be available for everyone.

In regard to trip-specific mode consideration, different feasibility assumptions per mode were 18considered taking advantage of the information derived from the Google API. Walking and cycling are 19considered for trips less than 3 km and 20km respectively (the maximum distance of trips where each 20have been chosen in the full dataset). Car is considered for trips of more than 50m distance to avoid the 21inclusion of short insignificant trips with close to 0 min travel time. Taxi is considered for trips of more 22than 50m and less than 81km (the maximum distance where taxi is the chosen mode in the full dataset). 23Bus is not considered for very short trips (where Google API suggests only walking segments) and only 24considered for trips with less than 3 transfers and with distance more than the minimum and less than the 25maximum distance of trips where bus is the chosen mode. Rail is not considered for very short trips 26(where Google API suggests walking only) and only considered for trips with less than 2 transfers and 27with distance more than the minimum distance of trips where rail is the chosen mode. Rail trips were 28allowed to include bus segments, since bus can be considered as an access/egress mode for rail, but that 29was not the case for the opposite scenario, i.e. using rail as an access/egress mode for bus.

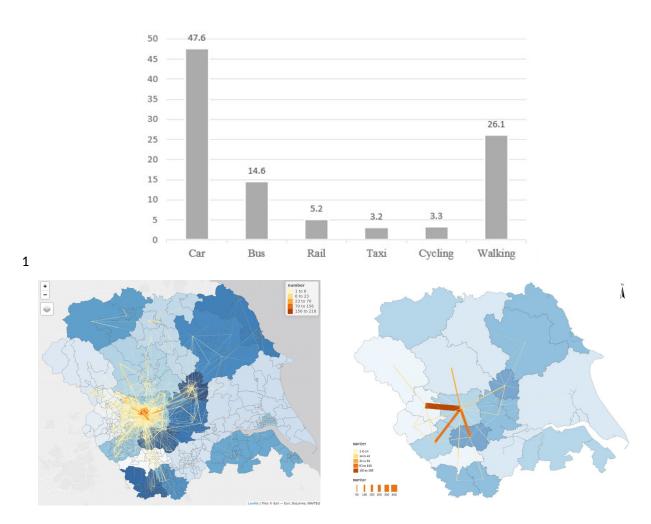
In terms of restrictions imposed by the earlier modes used in the tour, if car is the chosen mode 31 for the outbound trip, then the driver has no choice but to return the car to the starting location (home) 32 during the last trip of the tour making the rest of the alternatives unavailable. For that purpose, when car 33 is chosen for the first trip of a tour, the remaining alternatives are available for the remaining trips of the 34 tour except from the returning trip to home (**Figure 2**). Due to this constraint, those returning trips were 35 not included for mode choice modelling, since there is not any actual choice involved. On the contrary, 36 when bus, rail, taxi, cycling or walking is chosen for the first trip of the tour, car is available only for the 37 first trip, but not for the remaining trips of the tour (**Figure 2**). Finally, after considering all the 38 availability/consideration assumptions, the trips with only one available alternative were removed from 39 the following mode choice modelling.



2Figure :2 Tour-based mode availability

4 3.3 Descriptive statistics

5 The final dataset consists of 12524 trips made by 540 unique individuals. In **Figure 3**, the mode 6share and the OD pairs are depicted both for the trips starting and ending inside the region of Yorkshire 7 and those inside the Local Authority of Leeds. In general, the majority of trips start and finish inside 8Leeds and most of them are between Leeds and the neighbouring city of Bradford. In Table 1 the 9descriptive statistics of the sociodemographic features included in the dataset and the level of service 10attributes per available/considered mode, derived from the Google API, are presented. The sample 11includes more females than males and more individuals of higher education (from undergraduate level 12and above). Regarding the level of service attributes, the travel times, costs and distances are consistent 13with the expected values indicating an accurate estimation of those variables. Rail has the highest travel 14time and distance as it is considered a longer-distance mode compared to the rest and has the highest cost, 15since rail tickets are more expensive. Cycling and walking have the lowest average distance due to the 16require human effort. Bus travel distance is the lowest compared to the rest of mechanised modes, an 17indication that travelling longer distances by bus produces generally more discomfort than in the 18remaining alternatives. Taxi trips despite using the same network and routes as car trips have a lower 19travel time and distance, since the cost is significantly more compared to car.



2Figure 3: Mode share and OD pairs for trips inside the region of Yorkshire and the local authority 3of Leeds

5TABLE 1: Descriptive statistics of sociodemographic features and Level of Service attributes

Attributes (variable name)	Attribute levels	Number of observations	Percentage (%)
Soc	iodemographic features		
	Male	230	42.6
Gender	Female	310	57.4
	18-24	98	18.1
A	25-39	200	37.1
Age	40-59	201	37.2
	Above 60	41	7.6
	Employed	359	66.5
	Student	69	12.8
Occupation	Other (unemployed,	8	20.7
	retired, other		
	occupation)		
Household income	Below 50k	288	53.3
	Above 50k	198	36.7

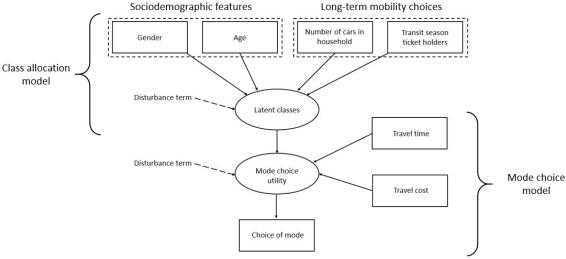
	Non-von-outous	Γ4	10.0
	Non reporters	54	10.0
	Lower (O-level, A-level, vocational)	156	28.9
Education	Higher (undergraduate, MSc, PhD)	384	71.1
	Single	165	30.6
Mr. St. I. e. e	Married	238	44.1
Marital status	Other (divorced, widowed, cohabiting)	137	25.4
Number of cars owned	mean	0.9	-
Number of bicycles owned	mean	0.8	-
Household size	mean	1.9	-
Transit season ticket holder	Yes	144	26.7
Transit season ticket notaer	No	396	73.3
Level	l of Service attributes		
Car travel time (min)	mean	17.1	-
Bus travel time (min)	mean	35.9	-
Rail travel time (min)	mean	60.5	-
Taxi travel time (min)	mean	15.8	-
Cycling travel time (min)	mean	22.8	-
Walking travel time (min)	mean	18.6	
Car travel cost (£)	mean	1.3	-
Bus travel cost (\mathfrak{L})	mean	2.6	-
Rail travel cost (£)	mean	10.4	-
Taxi travel cost (£)	mean	8.7	-
Car travel distance (km)	mean	10.7	-
Bus travel distance (km)	mean	8.3	-
Rail travel distance (km)	mean	21.2	-
Taxi travel distance (km)	mean	9.0	-
Cycling travel distance (km)	mean	6.3	-
Walking travel distance (km)	mean	1.5	-

2 4. RESULTS

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44.1 LCCM development (Model 1)

- Regarding the LCCM development, the basic form for the mode choice model component was 6developed first. Different model specifications were tested including the available level of service 7variables, namely *travel time*, *travel cost*, *in-vehicle travel time* and *access-egress time*, with the last two 8being specified only for bus and rail alternatives.
- For the latent class membership component, 2, 3, 4 and 5 classes have been tested with the aim of 10providing interpretable results and distinguishable individual behaviour among the classes. A model with 116 latent classes could not be estimated due to numerical issues and no further attempt was made to try 12models with additional classes. For the covariates included in the class allocation model, the requirement 13was to include variables that were statistically significant for at least one class and be able to provide a 14meaningful behavioural interpretation. Following that approach, the sociodemographic features of gender 15and age alongside the long-term mobility choices of number of cars in the household and the ownership 16of a transit season ticket for either bus or rail were included in the class allocation model. After the 17allocation of individuals into classes, the class-specific mode choice model is estimated using the ASCs, 18travel time and travel cost as independent variables. The final LCCM framework developed is illustrated 19in **Figure 4**.



2Figure 4 LCCM modelling framework

4 In order to select the preferred number of classes, a comparison was made among the estimated 5models with 2, 3, 4 and 5 classes in terms of LL, AIC, BIC and adjusted Rho-square Models consistently 6performed better with the addition of more classes as shown by the continuous improvement of all model 7fit statistics. Nonetheless, after the 3-class model the LL improved less with additional classes indicating 8diminishing returns in terms of model fit when choosing a more complex model specification. As a result, 9the 3-class model was selected as the most optimal segmentation of the sample population.

In **Table 2**, the modelling outputs of the class-specific and class allocation model for the 3-class 10 11LCCM are depicted and compared with the estimates of the unsegmented model. Further, as a measure of 12validation, the total VoT was calculated as the weighted average of the class-specific VoTs, excluding 13Class 1, due to a travel cost coefficient non-significantly different than 0. Moreover, the bus fare elasticity 14was estimated as the demand change for 1% ticket price increase. Those values were compared to the 15values suggested by the Transport Appraisal Guidance in the UK. The estimated VoT (9.9 £/hr) is very 16close to the suggested value (13.87 £/hr) (Department for Transport, 2014) although a little lower, and the 17estimated bus fare elasticity is -0.18 being at the lower bound of the suggested range of values (from-0.16 18to -0.65) (Dunkerley et al., 2018).

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Estimated parameters	Class 1	Class 2	Class 3	Unsegmented model
	Class alloca	ntion model		
Sample size percentage	51%`	28%	21%	-
Class-specific	-0.2070 <i>(-0.35)</i>	-0.7228 (-1.59)	-	-
constants				
	-			
Male (base)	=	=	-	-
Female	0.9559*** (2.73)	0.8117** (2.20)	-	-
Age 18-24 (base)	=	=	-	-
Age 25-39	0.2000 (0.40)	0.8678* (1.89)	-	-
Age 40-59	0.2179 (0.43)	0.7593 (1.50)	-	-
Age above 60	1.6649 (1.60)	2.6537** (2.42)	-	-
Number of cars	0.5604** (2.12)	-0.7986** (-2.43)	-	-

20Table 2: Estimated parameters for the class allocation, the class-specific (3-class model) and the 21unsegmented mode choice models

Transit season ticket holders	-1.177** <i>(-2.17)</i>	1.0875*** (2.61)	-	-								
	Class-specific mode choice model											
	Alter	native-specific constant	ts									
Car	-	-	-	-								
Bus	-4.1758*** (-10.63)	-0.2998 <i>(-0.73)</i>	-2.0197*** (-4.49)	-1.7177***								
				(-10.82)								
Rail	-1.7068*** (-4.53)	-2.0385*** (-3.56)	0.8816** (2.18)	-1.2837***								
				(-5.34)								
Taxi	-4.9392*** <i>(-13.46)</i>	-2.1033*** <i>(-4.45)</i>	-1.2008*** <i>(-1.75)</i>	-2.8984***								
				(-12.06)								
Cycling	-6.4372*** <i>(-11.48)</i>	-5.6561*** <i>(-6.95)</i>	-1.2855*** <i>(-3.94)</i>	-3.4101***								
				(-15.18)								
Walking	-1.3043*** (-5.12)	0.0450 (0.08)	0.5140 (1.40)	-0.3191* (-1.89)								
		el of Service attributes		_								
Travel time (min)	-0.061*** (-5.62)	-0.082** (-2.35)	-0.042*** (-6.17)	-0.0711***								
				(-8.02)								
Travel cost (£)	-0.138 (-0.42)	-0.335*** (-3.52)	-0.285*** (-4.99)	-0.2650***								
				(-7.51)								
		Validation measures										
VoT (£/hr)	_1	10.8	8.9	16.1								
Total VoT (£/hr)		9.9		16.1								
Bus fare elasticity		-0.18		-0.27								
		Fit statistics										
LL (final)		-4605.82		-5911.14								
AIC		9281.64		11836.27								
BIC		9541.88		11888.32								
Adjusted R-square		0.6901		0.6048								
Number of		540		540								
individuals												
Number of		12524		12524								
observations												

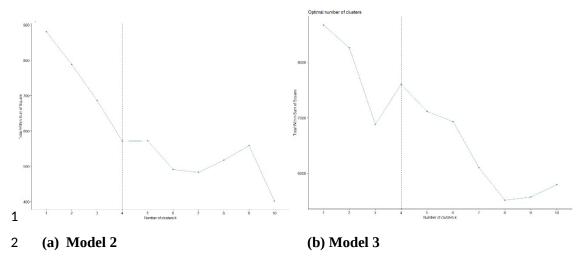
1(***, **, * Significant at the 99% (2.575), 95% (1.96) and 90% (1.645) confidence level, respectively)

44.2 K-modes/K-means clustering approach (Models 2 & 3)

In order to select the optimum number of clusters for K-means/K-modes, the "*Elbow*" method 7(Thorndike, 1953) was chosen to identify the cut-off point where no significant improvement on within-8the-cluster-sum-of-differences occurs anymore with the addition of more clusters. For the implementation 9of the "*Elbow*" method, the K-means/K-modes algorithm has been applied iteratively in the dataset for 10each case by specifying a different number of clusters each time, ranging from 1 to 10.

In Model 2 only sociodemographic data is included for clustering, namely the same covariates as 12in the LCCM, and the second stage a DCM is estimated per cluster. Since the included variables were all 13discrete, the K-modes algorithm was utilised. In Model 3, sociodemographic data and mobility-related 14information are included for clustering. The mobility data refers to the number of times each mode is 15chosen per individual. The K-means algorithm was implemented in that case, since there was a 16combination of discrete and continuous variables. At the second stage, a DCM is estimated per cluster.

²¹VoT for Class 1 could not be computed due to the non-significant travel cost parameter



3Figure 5 The "Elbow" method results

4 54.2.1 Model 2

The K-modes clustering algorithm was applied to the dataset and the sample was segmented into 74 clusters covering 53.4%, 31.6%, 10.8% and 4.2% of the sample, respectively. Comparing the 8sociodemographic percentages within each cluster with the unsegmented sample percentages, it was seen 9that members of Cluster 1, which covers more than half of the sample, have a high inherent preference for 10car. Cluster 2 has a distinct preference for car over the rest of the alternatives except for walking where 11there is a non-significant ASC. Furthermore, it is worth noting that this cluster has the highest VoT and 12bus fare elasticity. Cluster 3 has a non-significant predisposition for car over bus, rail and walking. 13Cluster 4, covering the smallest sample percentage, has a strong preference for car.

Regarding the model assessment, the combined total loglikelihood from the 4-cluster model 15provides a significant improvement over the unsegmented model as suggested by the LR-test. The 16validation measures of total VoT and bus fare elasticity are again within an acceptable range. The total 17LL (-5712.58), however, is significantly worse than the LCCM (-4605.82) indicating that there is still 18uncaptured heterogeneity in the two-stage Clustering-based model using only sociodemographic data.

23Table 3: Estimated parameters for the cluster-specific mode choice model of Model 2

Estimated	Estimated Cluster 1		Cluster 3	Cluster 4
purumeters	Alte	rnative-specific constan	ts	
Car	-	-	-	-
Bus	-2.0620*** (-8.44)	-1.6257*** (-6.12)	-0.3972 (-1.04)	-1.7740*** (-4.28)
Rail	-1.3368*** (-3.94)	-1.4795*** (-4.21)	-0.4955 <i>(-0.93)</i>	-1.4798*** (-2.72)
Taxi	-3.6579*** (-11.04)	-2.3727*** (-5. <i>7</i> 3)	-1.9202*** (-3. <i>06</i>)	-3.4811*** (-3.95)
Cycling	-3.2769*** (-12.28)	-4.9209*** (-9.39)	-3.2395*** (-3.89)	-3.1407** <i>(-2.57)</i>
Walking	-0.6367** (-2.32)	0.1383 (0.53)	0.2974 (0.71)	-1.2049** (-2.45)
	Le	vel of Service attributes		
Travel time (min)	-0.0591*** (-4.26)	-0.1027*** (-7.21)	-0.0735*** (-5.13)	-0.0561*** (-2.73)
Travel cost (£)	-0.2319*** (-5.52)	-0.3273*** (-5. <i>07</i>)	-0.2408** (-2.24)	-0.2805** (-2.47)
		Validation measures		
VoT (£/hr)	15.3	18.8	18.3	12.0

Bus fare elasticity	-0.26	-0.38	-0.12	-0.20						
Total VoT (£/hr)		16.	6							
Total bus fare	-0.28									
elasticity										
		Fit statistics								
LL (final)	-2983.54	-1659.70	-827.32	-242.04						
Total LL		-5712	2.58							
AIC	5981.08	3333.4	1668.64	498.08						
BIC	6028.74	3377.39	1705.09	527.89						
Adjusted R-square	0.6351	0.6357	0.4633	0.6156						
Sample percentage	53.4	31.6	10.8	4.2						
(%)										
Number of	288	178	50	24						
individuals										
Number of	6693	3959	1349	523						
observations										

34.2.2 Model 3

For the second clustering-based model, the 4-cluster model performed the best. The 4 estimated 5clusters cover 35.4%, 33.7%, 18.5% and 12.4%. The model outputs and the sociodemographic 6percentages of the clusters are presented in **Tables 4** and **6**, respectively. The striking feature is the 7difference in the socio-demographic characteristics of each cluster from Model 2.

9Table 4: Estimated parameters for the cluster-specific mode choice model of ML_choice_DCM

Estimated parameters	Cluster 1	Cluster 2	Cluster 3	Cluster 4
parameters	Altorno	ative-specific constants		
Car	Alterno	auve-specific constants		
Bus	-0.5170** (-2.54)	-3.3353*** (-11.70)	-0.7306** (-2.19)	-4.1227*** (-8.44)
Rail	-0.5871** (-2.28)	-1.9193*** (-3.43)	-0.0832 (-0.15)	-0.3457 (-0.48)
Taxi	-2.3488*** (-7.13)	-3.5611*** (-10.79)	-1.7874*** (-3.22)	-3.2076*** (-6.47)
Cycling	-2.1775*** (-7.33)	-3.8796*** (-10.97)	-2.7978*** <i>(-4.37)</i>	-6.9390*** (-7.25)
Walking	0.2429 (1.05)	-1.4793*** (-7.15)	1.7403*** (4.95)	-2.1422*** (- 5.55)
	Level	l of Service attributes		,
Travel time (min)	-0.0683*** (-5.89)	-0.0652*** (-6.25)	-0.0706*** (-3.08)	-0.0634*** (-5.02)
Travel cost (£)	-0.2056*** (-4.54)	-0.2717*** (-4.80)	-0.1851** (-2.56)	-0.4104*** (-7.05)
	Va	alidation measures		(112)
VoT (£/hr)	19.9	14.4	22.9	9.3
Bus fare elasticity	-0.16	-0.43	-0.22	-0.72
Total VoT (£/hr) Total bus fare elasticity		17.3 -0.33		
*		Fit statistics		
LL (final) Total LL	-3011.97	-1047.97 -5354.6	-1089.19 8	-205.55

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AIC	6037.94	2109.93	2192.39	425.1
BIC	6082.73	2154.36	2232.62	462.54
Adjusted R-square	0.4011	0.8033	0.561	0.8975
Sample percentage	35.4	33.7	18.5	12.4
(%)				
Number of individuals	305	147	62	26
Number of	4435	4219	2316	1554
observations				

1(***, **, * Significant at the 99% (2.575), 95% (1.96) and 90% (1.645) confidence level, respectively)

2

3 4.3 COMPARISON

4

As a comparison of the 3 models estimated it should be highlighted that the Model 1(LCCM) 6outperformed both clustering-based models in terms of model fit, thus resulting in more accurate 7estimates. Model 3 performed better than Model 2 which is expected given that it uses more inputs.

In terms of validation with the WEBTag values, all the values for the full models where close to 9the suggested values and within the suggested limits. The estimated VoT of LCCM was smaller than 10those of the two clustering-based models. On the other hand, Model 3 showed the highest VoT and bus 11fare elasticity. The other striking feature is the dissimilarity in the sociodemographic composition in the 12segments in the three models (**Table 6**) which denote how inherently different the three models are.

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1Table 5 Model comparison

Model		LCCM		ML_socio_DCM						WEBTag		
compariso	Class 1	Class 2	Class 3	Cluster	Cluster	Cluster	Cluster	Cluster	Cluste	Cluste	Cluster	values
n measures				1	2	3	4	1	r 2	r 3	4	
VoT (£/hr)	-	10.8	8.9	15.3	18.8	18.3	12.0	19.9	14.4	22.9	9.3	-
Bus fare	-	-	-	-0.26	-0.38	-0.12	-0.20	-0.16	-0.43	-0.22	-0.72	-
elasticity												
Total VoT		9.9			16	5.6		17.3				13.87
(£/hr)												
Total bus		-0.18		-0.28				-0.28 -0.33				
fare elasticity				ı.								to -0.65
Total LL		-4605.82			-571	2.58			-535	4.68		-

3Table 6: Sociodemographic percentages and qualitative assessment per model

Sociodemographic		LCCM			ML_soc	io_DCM			ML_choi	ice_DCM	
characteristics	Class 1	Class 2	Class 3	Cluster	Cluster	Cluster	Cluster	Cluster	Cluster	Cluster	Cluster
(sample				1	2	3	4	1	2	3	4
percentages/mean											
values)											
Sample size	0.51	0.28	0.21	0.53	0.32	0.11	0.04	0.35	0.34	0.19	0.12
percentage											
				Included so	ciodemogra	phic feature	S				
Male (0.43)	0.36	0.43	0.58	0.71	0.00	0.00	1.00	0.45	0.37	0.47	0.35
	(lower)	(average)	(higher)	(higher)	(lower)	(lower)	(higher)	(average)	(lower)	(average)	(lower)
Female (0.57)	0.64	0.57	0.42	0.29	1.00	1.00	0.00	0.55	0.63	0.53	0.65
	(higher)	(average)	(lower)	(lower)	(higher)	(higher)	(lower)	(average)	(higher)	(average)	(higher)
Age 18-24 (0.18)	0.16	0.17	0.25	0.13	0.23	0.38	0.00	0.18	0.07	0.53	0.00
	(average)	(average)	(higher)	(lower)	(higher)	(higher)	(lower)	(average)	(lower)	(higher)	(lower)
Age 25-39 (0.37)	0.35	0.41	0.36	0.27	0.69	0.00	0.00	0.40	0.36	0.31	0.23
	(average)	(higher)	(average)	(lower)	(higher)	(lower)	(lower)	(average)	(average)	(lower)	(lower)
Age 40-59 (0.37)	0.40	0.32	0.37	0.60	0.00	0.58	0.00	0.35	0.46	0.13	0.77
	(higher)	(lower)	(average)	(higher)	(lower)	(higher)	(lower)	(average)	(higher)	(lower)	(higher)
Age above 60	0.09	0.09	0.02	0.00	0.08	0.04	1.00	0.07	0.11	0.03	0.00
(0.08)	(average)	(average)	(lower)	(lower)	(average)	(lower)	(higher)	(average)	(higher)	(lower)	(lower)
Number of cars	1.1	0.6	0.8	1.0	0.8	0.6	1.1	0.8	1.3	0.04	1.5
(0.9)	(higher)	(lower)	(average)	(average)	(average)	(lower)	(higher)	(average)	(higher)	(lower)	(higher)
Transit season	0.12	0.53	0.28	0.23	0.12	1.00	0.21	0.39	0.10	0.15	0.08

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ticket Yes (0.27)	(lower)	(higher)	(average)	(lower)	(lower)	(higher)	(lower)	(higher)	(lower)	(lower)	(lower)
Transit season	0.88	0.47	0.72	0.77	0.88	0.00	0.79	0.61	0.90	0.85	0.92
ticket No (0.73)	(higher)	(lower)	(average)	(higher)	(higher)	(lower)	(higher)	(lower)	(lower)	(higher)	(higher)
				on included							
Household income	0.49	0.63	0.52	0.50	0.54	0.58	0.75	0.57	0.48	0.61	0.31
below 50k (0.53)	(lower)	(higher)	(average)	(average)	(average)	(higher)	(higher)	(higher)	(lower)	(higher)	(lower)
Household income	0.44	0.25	0.34	0.42	0.32	0.28	0.25	0.33	0.46	0.21	0.65
above 50k <i>(0.37)</i>	(higher)	(lower)	(average)	(higher)	(lower)	(lower)	(lower)	(lower)	(higher)	(lower)	(higher)
Household income	0.08	0.12	0.14	0.08	0.14	0.14	0.00	0.11	0.06	0.18	0.04
not reported (0.10)	(average)	(average)	(higher)	(average)	(higher)	(higher)	(lower)	(average)	(lower)	(higher)	(lower)
Education lower	0.29	0.33	0.24	0.33	0.17	0.36	0.46	0.31	0.33	0.13	0.19
(A-level, O-level,	(average)	(higher)	(lower)	(average)	(lower)	(higher)	(higher)	(average)	(average)	(lower)	(lower)
vocational) (0.29)											
Education higher	0.71	0.67	0.76	0.67	0.83	0.64	0.54	0.69	0.67	0.87	0.81
(undergraduate,	(average)	(lower)	(higher)	(average)	(higher)	(lower)	(lower)	(average)	(average)	(higher)	(higher)
MSc, PhD) (0.71)											
Marital status-	0.24	0.35	0.41	0.30	0.33	0.40	0.00	0.32	0.18	0.68	0.04
single (0.31)	(lower)	(higher)	(higher)	(average)	(average)	(higher)	(lower)	(average)	(lower)	(higher)	(lower)
Marital status-	0.51	0.35	0.40	0.49	0.38	0.24	0.75	0.43	0.54	80.0	0.85
married (0.44)	(higher)	(lower)	(average)	(higher)	(lower)	(lower)	(higher)	(average)	(higher)	(lower)	(higher)
Marital status-other	0.26	0.30	0.19	0.21	0.29	0.36	0.25	0.25	0.29	0.24	0.12
(divorced,	(average)	(higher)	(lower)	(lower)	(higher)	(higher)	(average)	(average)	(average)	(average)	(lower)
widowed,											
cohabiting) (0.25)											
Household size	2.0	1.6	2.3	2.0	1.9	2.1	1.0	1.9	1.8	2.3	2.4
(1.9)	(average)	(lower)	(higher)	(average)	(average)	(higher)	(lower)	(average)	(average)	(higher)	(higher)
Occupation	0.71	0.63	0.61	0.75	0.58	0.62	0.29	0.68	0.73	0.39	0.85
working (0.67)	(higher)	(average)	(lower)	(higher)	(lower)	(lower)	(lower)	(average)	(higher)	(lower)	(higher)
Occupation student	0.10	0.11	0.23	0.10	0.18	0.16	0.04	0.11	0.03	0.50	0.00
(0.13)	(lower)	(average)	(higher)	(average)	(higher)	(higher)	(lower)	(average)	(lower)	(higher)	(lower)
Occupation other	0.19	0.27	0.17	0.15	0.24	0.22	0.67	0.21	0.24	0.11	0.15
(unemployed,	(average)	(higher)	(average)	(lower)	(higher)	(average)	(higher)	(average)	(average)	(lower)	(lower)
retired, other											
occupation) (0.20)											

1 5. CONCLUSIONS

As a general conclusion, it should be highlighted that both sample segmentation approaches 3 resulted in models with better statistical fit than the unsegmented model. That alone is of great 4 significance from a policy perspective. The general practice of estimating and applying unsegmented 5 behavioural models to real-world projects could have adverse effects in project valuation and in creating a 6 model shift to more sustainable modes, since they are based on biased estimates. Incorporating latent 7 lifestyles into a behavioural model can provide an insight on the individuals' subconscious taste 8 preferences leading to better planned policy initiatives. Regarding the comparison of the models 9 estimated, the LCCM outperformed the clustering-based models by a large margin. From the two 10 clustering-based models, Model 3 (which included both socio-demographics and choices) resulted a better 11 model fit than the one that uses only socio-demographics. Furthermore, significant differences were 12 observed in the composition of the segments in the three methods.

Nonetheless, the current study was also subject to certain limitations. The initial goal during 14model development was to estimate a mixed LCCM model and mixed MNL models for the clustering-15based approach to capture heterogeneity both across classes/clusters and among individuals within the 16same class/cluster. That was not possible, however, due to numerical issues in mixed LCCM. 17Specifically, the presence of a non-significant travel cost parameter for Class 1 resulted in errors during 18the calculation of the covariance matrix, possibly because no further heterogeneity could be captured for 19an already insignificant parameter. Furthermore, there are more advanced clustering algorithms, such as 20the "Soft" or "Fuzzy" K-means/K-modes which performs a probabilistic clustering by allocating a data 21point to a cluster with a certain probability or weight. As a next step, the performance of those more 22advanced "Soft" K-means/K-modes algorithms is going to be assessed, which is believed to provide 23better results than the traditional algorithms tested in the current study.

Even in their current form, however, the findings are expected to provide guidance to researchers 25and practitioners in choosing the most efficient method to capture the taste heterogeneity among the 26segments of the travelers.

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34AUTHOR CONTRIBUTIONS

The authors confirm contribution to the paper as follows: study conception and design: PT, CC 36& SH; data collection: SH, PT & CC; analysis and interpretation of results: PT, CC & SH. All authors 37reviewed the results and approved the final version of the manuscript.

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