Capturing zero-price effects in stated choice surveys and implications for willingness-to-pay computation

Jeff Tjiong, Stephane Hess, Thijs Dekker, Marek Giergiczny, Manuel Ojeda-Cabral, Mikołaj Czajkowski

Abstract

1 Introduction

Stated choice (SC) surveys are used extensively to forecast demand and capture welfare effects of policy changes (see Louviere et al., 2000, Ben-Akiva et al., 2019). While it is a common practice to apply random utility maximisation (RUM) models for analysing choice behaviour based on SC data (McFadden, 1986, McFadden et al., 1986, Carson and Louviere, 2011), there has been a growing interest in alternative behaviour phenomena in the choice modelling literature (Hess et al., 2018, Hensher, 2019).

Amongst these alternative behavioural phenomena, a well-established notion in behavioural economics is that individuals tend to get over-attracted to free alternatives. This psychological mechanism was coined as the zero-price (ZP) effect (Shampanier et al., 2007, Ariely, 2008). The ZP effect has been demonstrated in many different forms. Some examples include the choice of chocolate brands (Shampanier et al., 2007), hotel room bookings in a two-component setting where the breakfast could be free (Nicolau and Sellers, 2012), pseudo-free offers where nonmonetary payments (e.g. time, personal information, privacy) are included as cost components (Dallas and Morwitz, 2018), access to e-services (e.g. music/video streaming services) that could be free (Hüttel et al., 2018), and premiums to health insurance which are subsided by public money (Douven et al., 2019).

Despite a wealth of evidence of the ZP effect, the choice modelling literature has paid very little attention to this phenomenon. Some exceptions include Hess et al. (2008), Hess and Rose (2009), and Hess and Beharry-Borg (2012). The lack of explicit treatment of ZP effect in discrete choice applications creates significant risks in producing biased parameters and welfare measures, which could lead to inferior policy recommendations. Namely, not accounting for the ZP effect would lead to an overestimation of the cost sensitivity and hence under-estimated willingness-to-pay (WTP) measure.

The lack of attention to the ZP effect in the environmental economics and health literature is particularly worrying since 'free' status quo (SQ) alternatives are at the heart of most SC surveys and form the basis of contrasting the (policy) 'interventions'. Indeed, researchers have conducted in-depth analyses of the over-reaction towards the SQ alternative, namely, the SQ effect (Samuelson and Zeckhauser, 1988), including the understanding of the behavioural rationale (Meyerhoff and Liebe, 2009, Adamowicz et al., 1998, Zhang and Adamowicz, 2011), econometric issues regarding model specifications for the SQ effects (Scarpa et al., 2005, Oehlmann et al., 2017) and impacts of SQ effect to the welfare analysis (Adamowicz et al., 2011). However, despite the perfect confounding between the ZP effect and the SQ alternative, the ZP effect has rarely been mentioned as a possible behavioural cause for the SQ effect (see discussion in Hess and Beharry-Borg (2012)).

This paper aims to separate the ZP and SQ effects in order to assess how much of the observed preference for maintaining the SQ is influenced by the disproportional attraction to the zero cost only. This is accomplished by re-framing the SQ context such that both free and non-free SQ alternatives are presented to respondents. From a policy perspective, understanding the extent to which ZP effect can affect people's choices is important. In many empirical contexts, maintaining the

SQ will be associated with (positive) costs to prevent deterioration. This is particularly relevant to the valuation of environmental goods or health policies. Placing even a minimal cost for maintaining the SQ can significantly reduce its attractiveness in the presence of the ZP effect. Ignoring such effect may lead to under-estimation of the acceptability of the designed policy intervention. However, due to perfect confounding between the ZP and the SQ effect in most stated choice surveys, marginal WTP estimates may only be effected to a limited extent.

This paper also identifies further complications in capturing the ZP effect that arise where respondents exhibit non-linear sensitivity to cost (Daly, 2010, Rich and Mabit, 2016). First, it is common to see a price gap between zero cost and the next cost level in experimental designs. This also implies, however, an inadequate number of trade-offs near zero cost levels that are essential for the identification of the ZP effect. Namely, the detection of the disproportionate increase of cost sensitivity from zero cost to near zero cost as exhibited by the ZP effect. Where a price gap exists by design, it becomes difficult for choice models to distinguish between the ZP effect and the non-linearity in utility, as both specifications could resemble the higher cost sensitivity than average perceived by respondents at small costs. To ensure that the ZP effect and the non-linear sensitivity can be separated, sufficient number of small costs are incorporated in our design. Second, it is known that non-linearity in cost sensitivities may erroneously be picked up as ZP effects with linear sensitivities, where the reverse also applies (Hess et al., 2011). Therefore, flexible model specifications are also tested in estimation to minimise the risks of obtaining biased parameter estimates and welfare measures due to utility misspecification.

The remainder of the paper is structured as follows. Section 2 describes the experimental setup. Section 3 outlines the research methodology. Section 4 summarises the model results from the SC data. Lastly, Section 5 discusses policy implications and concludes.

2 Experimental setup

2.1 Design overview

Our experiment draws on SC data collected from 302 students at the University of Warsaw (Poland) in late 2017. Despite our interest in framing our elicitation in the environmental or health economics context, we instead focus on the elicitation of WTP for 4G LTE data packages from students. This is based on the consideration that it is critical in SC elicitations for survey subjects to be familiar with the products or designed policies and are expected to make similar decisions in the future (Ben-Akiva et al., 2019). Where applicable, we describe how our experimental setup can be applied to the environmental or health economics context in latter sections.

Respondents are asked to choose between retaining the free campus-wide Wi-Fi service (i.e., the SQ alternative) or to purchase a 4G LTE data package which allows access to high speed mobile data beyond the school campus by using a USB dongle. 3 attributes are varied amongst choice tasks: monthly costs of the 4G LTE data package, monthly data download limit, and the number of devices that can share bandwidth simultaneously. The cost levels of the 4G LTE data package are set to be lower than the commercial packages typically offered by the major mobile network operators to create incentives for students to consider the mobile data packages. Prior to the stated choice tasks, respondents are required to answer a few basic questions concerning their current internet usage experience, specifications of existing mobile data packages etc.

This experiment is set out to examine the impact of ZP effects on utility via 3 different treatments. The first treatment (SP1) mimics a common format of choice sets in environmental and health economics which includes a SQ alternative with zero price and two experimentally designed alternatives (i.e., the standard '2+*SQ*' format as described in Ferrini and Scarpa (2007)). This is also where ZP and SQ effects are perfectly confounded. In the second treatment (SP2), both free and non-free SQ alternatives are allowed for separation of the ZP effects from the SQ effects. This is applicable in the environmental and health economics context by assuming an out-of-pocket cost (e.g. entrance fee, tax) that is required to maintain the otherwise deteriorating environment or health condition. Finally, the SQ alternatives are dropped in the third treatment (SP3) to focus on the trade-offs between alternatives where zero or near-zero costs are included. Adequate small cost levels are incorporated for better separation of the non-linearity in cost sensitivity and ZP effect econometrically.

These 3 treatments are presented sequentially to each respondent to allow us to observe the variation in terms of the finding of the ZP effect and the resulting WTP measures between treatments. Respondents are required to answer 26 choice tasks in total. SP1 and SP2 are blocked into 2 sets of 8 choice sets. SP3 consists of 30 choice tasks that are blocked into 3 sets of 10 choice tasks. All 3 treatments are created based on a Bayesian *D*-efficient design. Priors are obtained from pilot surveys and are assumed to be normally distributed. **Table 1** provides an overview of the attributes and levels set out for each treatment.

Table 1 – Overview of attributes

Treatment	Alt	Monthly fee (zł)		4G data limit (GB/month)	4G data accessibility (# of devices)
		Wi-Fi (campus)	4G LTE data		
SP1	SQ	0	-	-	-
	Alt 2/3	0	5 / 10 / 15 / 20 / 30 / 40	3 / 5 / 10 / 20	1 / 3
SP2	SQ	0 / 1 / 3	-	-	-
	Alt 2/3	0 / 1 / 3	5 / 10 / 15 / 20 / 30 / 40	3 / 5 / 10 / 20	1 / 3
SP3	Alt 2/3	-	0 / 1 / 2 / 3 / 5 / 8 / 10 / 20 / 30 / 40	3 / 5 / 10 / 20	1 / 3

2.2 Individual treatment

SP1 – Zero cost SQ alternatives

All SQ alternatives in SP1 are assumed to be free of charge. The SQ is framed in the way that students would rely on the free Wi-Fi connection already provided by the university. Students are informed prior to the presentation of valuation questions that the university would offer all students with both free a SIM card and a USB modem (the size of a dongle/pendrive) with Wi-Fi connectivity for any 4G LTE broadband package chosen, thus enabling the use of high-speed data transfer both within and outside the university. The browsing speed offered by the free Wi-Fi service is known to students to be slower compared to the 4G LTE data connection.

SP2 – Zero and non-zero cost SQ alternatives

Some variations in cost for the SQ alternatives are needed for separating the ZP and SQ effects. Hence, a small charge, either in 1 Polish Zloty (zł) or 3zł for the Wi-Fi access (i.e., the SQ alternative), is framed as a mandatory policy. These small costs required to maintain current service levels of the computer network provision are presented in 10 out of 16 choice tasks in SP2 while the remaining 6 choice tasks offers free SQ alternatives. Respondents are given the instruction prior to the valuation questions that for choosing any of the 4G LTE data packages, the total costs will then include both the minimal charges for the campus-wide Wi-Fi connectivity and the costs of the 4G LTE data package. This implies that the minimal price gap between the 4G LTE data package and the SQ alternatives remains at 5zł, which is the same as in the SP1 (also see **Table 1**).

SP3 – No SQ alternative

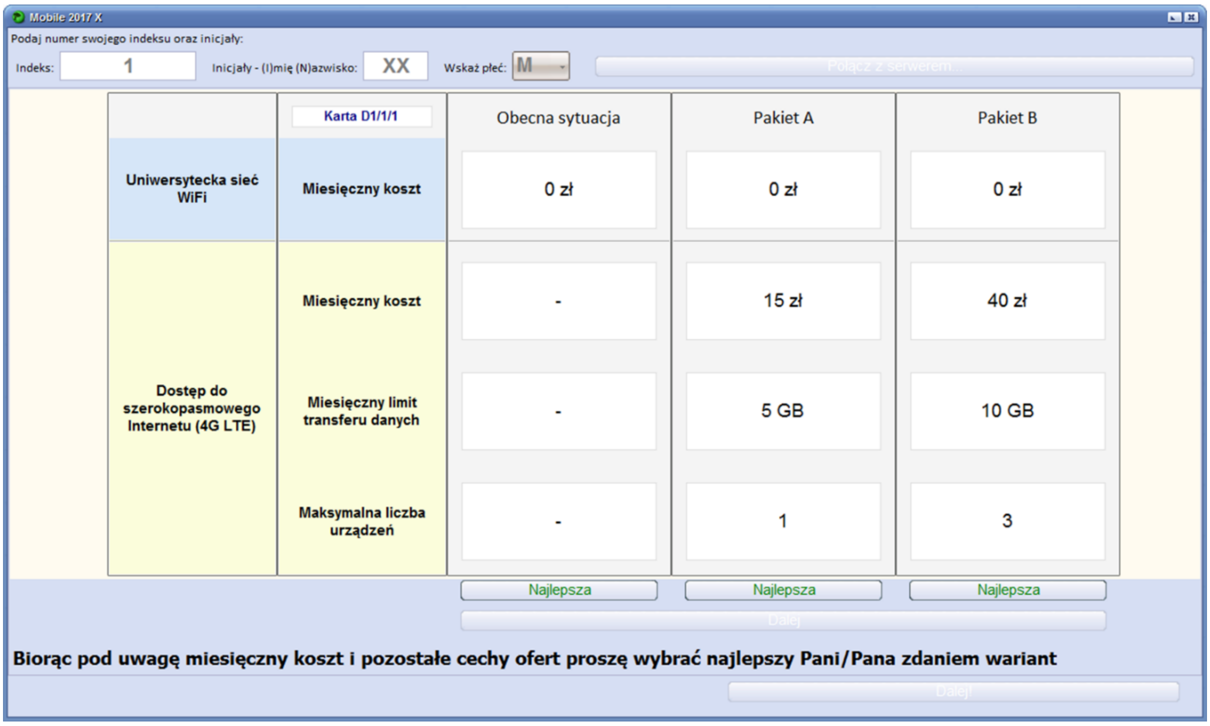
This treatment presents binary choices between two 4G LTE data packages. The removal of the SQ alternative is designed to ensure that no status quo effect would come into play. Both zero and near-zero cost levels are introduced in this treatment. Respondents are asked to choose between a free and non-free alternative in 3 out of 10 choice tasks and to choose between the non-free 4G LTE data packages in the rest of the 7 choice tasks. Small cost levels (1 zł, 2 zł and 3 zł) are introduced in this treatment to allow detection of changes of the marginal cost sensitivity curve at near-zero cost levels in more confidence such that we can better distinguish the ZP effect from non-linearity in cost sensitivity. Despite there are only two choices presented in SP3, we retain the numbering of the two non-free alternatives for the 4G data package in SP1 and SP2 for consistency in summary of model results. Namely, 'alternative 2' referring to the first 4G data package and 'alternative 3' which refers to the second 4G data alternative.

Survey undertaking

The SC experiment is carried out through a survey app provided to respondents (see **Figure 1**). A follow-up question is prompted to respondents automatically whenever non-trading behaviour is detected from respondents after completing all the choice tasks in the first two treatments. This mainly allows us to understand the rationale behind respondent sticking with a given option. Approximately 10% of respondents in SP1 exhibit non-trading behaviour towards the SQ option. The majority of them indicated that they are satisfied with the existing services and hence are not related to other contextual issues (e.g., protest against the university policy to start charging for the campus-wide Wi-Fi provision)[1]. A pilot survey was carried out prior to the main survey undertaking.

[1] In SP1, 11% of respondents are SQ non-traders in SP1, 9% of respondents are satisfied with their current package, with the rest of the 2% respondents indicated that they cannot switch operator anytime soon. In SP2, 9% of respondents are SQ non-traders, only 1% of respondents are against charging of the Wi-Fi connection.

Figure 1 – Sample SP1 choice task in the SC survey app



3 Methodology

3.1 Model specification

The SC data collected are analysed using a standard RUM-based choice model, where the indirect utility U_{jnt} obtained for an individual n (with $n = 1, \dots, N$) for alternative j (with $j = 1, \dots, J$) in choice task t is decomposed into a deterministic component V_{jnt} and a random component ϵ_{jnt}:

$$U_{jnt} = V_{jnt} + \epsilon_{jnt} \tag{1}$$

It is assumed that ϵ_{jnt} follows an extreme value distribution across alternatives. Assuming linear attribute sensitivities for the base specification, the deterministic component of the utility of alternative j applied for all three treatments can be generalised by:

$$V_{jnt} = \delta_j + \delta_{ZP}\left(Cost_{jnt} == 0\right) + \beta_{cost}Cost_{jnt} + \beta_{dlim}Dlim_{jnt} + \delta_{dev_j}(Dev_{jnt} == 3) \tag{2}$$

where δ_j is a constant associated with alternative j to capture the average effect on utility due to the tendency of choosing a particular alternative. This is normalised to zero for a base alternative. δ_1 captures the SQ effects in SP1 and SP2 as SQ alternatives are the left-most alternatives (i.e. $j = 1$); δ_{ZP} is a dummy variable estimated in the case where the alternative j is a zero-price alternative; β_{cost} is the marginal utility associated with the total cost for alternative j, $Cost_{jnt}$, which includes the costs for both the 4G LTE data package and the Wi-Fi, expressed in Polish złoty (zł)[2]; β_{dlim} is the marginal utility associated with the data limit of the 4G LTE data package, $Dlim_{jnt}$, expressed in gigabytes (GB) per month; δ_{dev_j} is a dummy variable estimated when the alternative j allows up to 3

[2] Since the cost items are presented to respondents separately, we also take into consideration that respondents may respond to the costs of Wi-Fi and 4G LTE data packages differently (i.e. different cost sensitivities). However, test results using data SP2 and SP3 indicate that separate cost sensitivities cannot be identified.

devices to access the 4G LTE mobile data ($Dev_{jnt} = 3$). As we will discuss in latter section, some model specifications allow departures from the base linear-in-attribute specification to include the possibility of non-linear sensitivities to data limit and/or cost attribute by introducing the non-linear transformation of attributes.

This paper focuses on explaining the unobserved part of the utility deterministically by the association with ZP effect, which in our view is more preferable in explaining behavioural patterns, and lowers the further requirements for incorporating random taste heterogeneity (see Train, 2009, Hess et al., 2005, Hensher and Greene, 2003) or more complicated error correlation structure (Scarpa et al., 2005).

3.2 Modelling non-linearity

Box-Cox transformation (Box and Cox, 1964) is adopted to incorporate the possibility of non-linear sensitivity for the continuous cost attributes. This is a common approach to non-linear treatment (Daly, 2010, Gaudry et al., 1989, Rich and Mabit, 2016), which applies a flexible functional form that "detects" the degree of non-linearity within the SC data. To conform to the notion of diminishing cost sensitivity (i.e. a damping effect), the non-linearity of the cost attribute is bounded by a linear relationship when $\lambda = 1$ and a log-transformed function when λ approaches 0. The transformation of the total cost attribute for alternative j for choice task t, $Cost_{jnt}$, is given by:

$$Cost_{jnt} = \begin{cases} \dfrac{Cost_{jnt}^{\lambda} - 1}{\lambda}, & \lambda \neq 0;\ Cost > 0 \\ \ln(Cost_{jnt}), & \lambda = 0;\ Cost > 0 \end{cases} \tag{3}$$

It is also evident in some earlier model results that respondents show decreasing sensitivity to data limit of the 4G LTE data package presented. Given the non-linearity in sensitivity to data limit, we applied the log-transformation to the data limit attribute, $\ln(Dlim_{jnt})$, for all model specifications.

Willingness-to-pay measures are generated for the data limit, which represents the marginal rate of substitution between 4G LTE data limit and cost. This is given by the ratio of the partial derivative of the indirect utility function with respect to data limit to the partial derivative with respect to cost. Since a log-transformation of data limit attribute is applied to all model specifications, the partial derivative with respect to data limit becomes $\beta_{dlim}/Dlim$. The partial derivative with respect to cost, however, varies depend on specifications. When cost sensitivity is specified linearly, then the WTP for data limit becomes $\frac{\beta_{dlim}}{Dlim}/\beta_{cost}$. When the possibility of non-linearity in cost sensitivity is incorporated in the utility formulation using a Box-Cox transformation, the WTP for data limit is given by:

$$WTP_{dlim} = \frac{\partial V/\partial Dlim_{LN}}{\partial V/\partial Cost_{BC}} = \frac{\beta_{dlim}/Dlim}{\beta_{cost}(Cost)^{\lambda-1}} \quad where\ \lambda \geq 0;\ Cost > 0;\ Cost_{BC} \sim BoxCox(Cost; \lambda) \tag{4}$$

Equation 4 shows that the WTP calculation depends on both the monthly cost and the data limit allowance. A set of reference measures is thus needed for comparing WTP estimates across scenarios, as opposed to simply comparing to an average WTP at sample level in the case where marginal utilities are specified linearly. 2 representative data limits at 5GB and 20GB and 3 representative cost levels at 3zł, 10zł, and 30zł are chosen for summary of model results. This results in 6 reference WTP measures that are compared across all models in this paper. Delta method is implemented to obtain the estimates of error in the derived WTP measures (Daly et al., 2012). The computation of standard errors for each of the three cost specifications that vary in the value of λ are outlined in detail in **Appendix A**.

A Monte Carlo (MC) simulation is also carried out during the model development stage to reassure that a range of non-linearity in cost and data limit, and the ZP dummy assumed in the true data generating process (DGP) can be retrieved econometrically in model estimation when the utility formulation is specified correctly. The simulation approach also provides useful insights into the risks of biased WTP due to model misspecification (Williams and Ortúzar, 1982). Our test model estimation using simulated data indicates that the ZP dummy can capture some cost non-linearities when cost is misspecified linearly, and conversely, the marginal utility coefficients can capture some of the behavioural effects for not having constants. This is in line with the findings in Hess et al. (2011) which calls for the use of flexible functional form while capturing effects that are not modelled by marginal utilities in utility.

4 Empirical results

4.1 SP1 – Zero cost SQ alternatives

We start off with a basic utility specification that is commonly applied in practice. Model SP1-L1 relies on two alternative specific constants (ASC), δ_{alt1} and δ_{alt2}, to capture the average of all the effects on utility that are not modelled for an alternative. Cost sensitivity is specified linearly. All parameter estimates presented in **Table 2** are statistically significant at 95% confidence level. The SQ constant, δ_{alt1}, is estimated at 0.737 and takes on a positive sign, which is consistent to the notion of SQ effect as respondents show strong preference to remain at the SQ. Since the ZP and SQ constants are perfectly confounded by construct, we cannot exclude the possibility that δ_{alt1} also captures certain degree of the ZP effects, if any. Sensitivity to the log-transformed data limit, β_{dlim}, is estimated at 1.074, which is highly significant with a robust *t*-ratio of 16.82. This supports the proposition of a decreasing sensitivity for higher data limit. The positive and statistically significant dummy variable of δ_{mdev} is estimated at 0.412. This reflects the preference for allowing access of the 4G LTE data package from multiple devices, which is in line with our expectation.

WTP for 4G LTE data derived from the model SP1-L1 varies depending on the data limit level. This results from the first derivatives of both the non-linear marginal utility of data limit and the linear marginal cost utility. Results show that respondents are willing to pay 2.21zł for each additional GB of 4G LTE data for a package that comes with 5GB of data limit. A significantly lower WTP for data limit is estimated at 0.552zł/GB when a higher data limit of 20GB is offered. This 75% decrease in WTP is equivalent to the relative ratio in data limit (i.e., 5GB vs. 20GB), as the WTP is inversely proportional to the size of the data limit. Robust *t*-ratios for the 6 reference WTP values remain the same as both the standard error (see **Appendix A**) and the WTP (see **Section 3.2**) are inversely proportional to the data limit. The calculation of the *t*-ratio is thus not affected by the variation in data limit (presented either in the reference data limit of 5GB or in 20GB in **Table 2**) as the effect of the size of the data limit is cancelled out.

Table 2 - Estimation results for SP1 and SP2

		SP1 - L1 Linear Cost		SP2 - L1 Linear Cost		SP2 - BC1 Box-Cox~(Cost)	
Respondents		302		302		302	
Obs		2416		2416		2416	
Final LL		-2295.33		-2173.93		-2165.67	
AIC		4600.66		4359.85		4345.34	
Adj. ρ^2		0.133		0.179		0.181	
Parameter estimates		est.	rob. t-rat(0)	est.	rob. t-rat(0)	est.	rob. t-rat(0)
ZP (δ_{ZP})		-	-	0.246	3.38	0.083	1.06
SQ_{alt1} (δ_{alt1})		0.737	4.84	0.531	3.03	-1.175	-1.59
ASC_{alt2} (δ_{alt2})		0.085	1.97	0.025	0.53	0.067	1.40
$Cost_{Linear}$ (β_{cost})		-0.097	-16.74	-0.086	-20.36	-	-
$Cost_{Box\text{-}Cox}$ (β_{cost})		-	-	-	-	-0.873	-2.05
$Lambda_{Box\text{-}Cox}$ (λ)		-	-	-	-	0.261	1.63
Data $Limit_{log}$ (β_{dlim})		1.074	16.82	0.904	15.22	1.038	14.10
Multi-Access (δ_{mdev})		0.412	6.78	0.396	5.54	0.327	4.53
WTP (zł/GB) at reference 4G LTE data limit & total cost							
Data Limit (GB)	Cost (zł)	est.	rob. t-rat(0)	est.	rob. t-rat(0)	est.	rob. t-rat(0)
5	3	2.207	17.77	2.110	14.30	0.535	2.04
5	10	2.207	17.77	2.110	14.30	1.304	2.16
5	30	2.207	17.77	2.110	14.30	2.937	2.19
20	3	0.552	17.77	0.528	14.30	0.134	2.04
20	10	0.552	17.77	0.528	14.30	0.326	2.16
20	30	0.552	17.77	0.528	14.30	0.734	2.19

4.2 SP2 - Zero and non-zero cost SQ alternatives

Linear cost specification

SP2 is devised to disentangle the ZP and SQ effects by allowing both zero and non-zero costs for the SQ alternatives. Largely based upon the linear-in-cost specification adopted in SP1, the ZP dummy, δ_{ZP}, is now split from the SQ constant, δ_{alt1}, that merely captures the preference of remaining at the SQ even when the SQ alternative is not free. Model results (SP2-L1) are presented in **Table 2**. Most parameters are statistically significant at 95% confidence level, except for the constant associated with the middle alternative (i.e. alternative 2). Both the ZP and SQ constants are statistically significant. The result suggests that the extra attractiveness to the SQ alternatives that are not captured by the marginal utilities can be partly explained by the ZP effect, rather than the preference of remaining at SQ at its own. The ZP and SQ constants are estimated at 0.246 and 0.531. Taking ratio of these two constants finds that the SQ effect is approximately 2.2 times the impact of the ZP effect. In other words, Model SP2-L1 implies that the impact of the tendency to remain at the SQ options not related to the attractiveness to the ZP can be over-estimated by 46% if the confounding of the SQ and ZP effects are not disentangled.

Comparing against the model results from model SP1-L1, WTP estimates reduce slightly by 4.4% from 2.21zł/GB and 0.55zł/GB for the alternatives with either 5GB or 20GB data allowance, respectively in model SP1-L1, to 2.11zł/GB and 0.53zł/GB once non-linear cost sensitivities are included in model SP2-L1. The differences in WTP between the two treatments are not statistically significant (*t*-ratio of 0.50), which indicates that the impacts on the contextual difference (e.g. protest behaviour) between SP1 and SP2, namely, the introduction of minimal yet new charges on the Wi-Fi services, do not appear to affect computation of the WTP for 4G LTE data packages.

Non-linear cost specification

A flexible non-linear formulation that adopts a Box-Cox transformation of costs, is also tested to examine whether the presence of the SQ and ZP effects implied by model SP2-L1 are more likely to be contributed by the real behavioural effect, or alternatively, are artefacts of the misspecification of the cost sensitivity. Estimation results from this model SP2-BC1 are summarised in **Table 2**.

The final log-likelihood improves from -2,173.93 in model SP2-L1 to -2,165.67 in model SP2-BC1 simply by incorporating non-linearity in cost sensitivity. The Box-Cox transformed cost coefficient is estimated at -0.873 and is statistically significant at 96% confidence level (robust *t*-ratio of 2.05). All As opposed to the findings from the model SP2-L1, the ZP and SQ constants are not statistically significant at 95% confidence interval, indicating that the systematic difference in preference of the zero cost and remaining at the SQ is not significantly strong in model SP2-BC1. This supports the proposition that the ZP and SQ effects captured in the linear-in-cost model in SP2 are partly capturing non-linearities in cost sensitivity due to the utility misspecification. It is noted that this finding is based upon the better fit to data shown by specifying cost sensitivity non-linearly. That said, our finding is in line with both past literature (Hess et al., 2011) and our initial simulation runs that highlight the risk of obtaining biased estimates for the constant where the utility formulation is misspecified.

WTP also gives very different picture to the linear-in-cost model as WTP is much smaller at small cost levels, and vice versa for more expensive alternatives. When the cost level is set at 3zł, the WTP reduces by 75% from 2.11zł/GB and 0.53zł/GB in model SP2-LC1 for the alternatives with 5GB and 20GB data limits, to 0.54zł/GB and 0.13zł/GB in model SP2-BC1, respectively. This reflects the damping effect in cost sensitivity facilitated by the Box-Cox transformation, which gives higher cost sensitivity at small cost levels compared to the average cost sensitivity across all cost levels estimated by the linear-in-cost specification. Conversely, the WTP for data package offered at 30zł increases by 39% from 2.11zł/GB and 0.53zł/GB for the alternatives with 5GB and 20GB allowance in the linear-in-cost model SP2-LC1, to 2.94zł/GB and 0.73zł/GB in model SP2-BC1, respectively.

Overall, model results in SP2 suggests that there is no significant ZP effect or SQ effect detected in this treatment after the use of flexible utility formulation to verify the validity of the presence of SQ and ZP effects. That said, there remains a price gap between zero cost and the first cost level of 5zł for the 4G LTE data packages. Therefore, it is still inconclusive whether the non-linearity and ZP effect can be disentangled fully especially near the zero cost. This issue is dealt with explicitly in SP3.

4.3 SP3 – No SQ alternative

Linear cost specification

Binary choices are presented in SP3 with the possibility of zero cost in one of the two alternatives available. Respondents are not subject to any SQ effect by design as the SQ alternatives are excluded in SP3. This avoid any confounding between ZP and SQ effects entirely. Very small costs are presented to ensure that the ZP effect can be distinguished from the non-linearity in cost sensitivity near the zero price. This treatment thus represents the best 'test-bed' for capturing the ZP effect amongst the 3 treatments. Similar to the previous treatments, we first specify a basic linear-in-cost model with a dummy variable to capture any potential ZP effect. Results for this model SP3-L1 are presented in **Table 3**. All parameters are statistically significant at 95% confidence level and in the expected sign. The ZP dummy is estimated at 0.501 (robust *t*-ratio of 5.78), which appears to have captured the ZP effect which is statistically significant. The WTP values are reduced by 22% from the linear-in-cost model in SP2 (model SP2-L1), from 2.11zł/GB and 0.53zł/GB for data limit provided at 5GB and 20GB in SP2-L1, to 1.64zł/GB and 0.41zł/GB in SP3, respectively. The lower WTP for 4G LTE data package estimated in SP3-L1 can be attributed to the introduction of the small costs in SP3. This finding is verified by a test model that applies the same specification as with SP3-L1 but with trade-offs involving small costs removed from the model estimation, which produces WTP values that are not statistically different to those obtained in model SP2-L1. Clearly by enriching the experimental design with small costs, we allow the model to identify some high cost sensitivity perceived by respondents at small costs and hence reduce the risk over-estimating the WTP.

Table 3 – Estimation results for SP3

		SP3-L1 Linear Cost		SP3-BC1 Box-Cox~(Cost)	
Respondents		302		302	
Obs		3020		3020	
Final LL		-1553.59		-1509.07	
AIC		3117.18		3030.14	
Adj. ρ^2		0.2554		0.2762	
Parameter estimates		est.	rob. t-rat(0)	est.	rob. t-rat(0)
ZP (δ_{ZP})		0.501	5.78	0.362	3.84
ASC_{alt2} (δ_{alt2})		0.108	2.60	0.073	1.73
$Cost_{Linear}$ (β_{cost})		-0.121	-19.32	-	-
$Cost_{Box\text{-}Cox}$ (β_{cost})		-	-	-0.535	-7.74
$Lambda_{Box\text{-}Cox}$ (λ)		-	-	0.469	9.90
Data $Limit_{log}$ (β_{dlim})		0.992	13.83	1.181	14.91
Multi-Access (δ_{mdev})		0.299	5.53	0.334	6.04
WTP (zł/GB) at reference data limit & cost					
Data Limit (GB)	Cost (zł)	est.	rob. t-rat(0)	est.	rob. t-rat(0)
5	3	1.642	16.50	0.791	7.55
5	10	1.642	16.50	1.500	7.87
5	30	1.642	16.50	2.689	7.97
20	3	0.410	16.50	0.198	7.55
20	10	0.410	16.50	0.375	7.87
20	30	0.410	16.50	0.672	7.97

Non-linear cost specification

We examine the validity of the findings with respect to the ZP effect from model SP3-L1 by applying a Box-Cox transformation of costs in Model SP3-BC1. As shown in **Table 3**, the final log-likelihood improves significantly from -1,553.59 to -1,509.07, which indicates that the specification of non-linear cost sensitivity better fits the SC data. All parameters are statistically significant at 95% confidence level except for the ASC controlling for the preference towards the left-most alternative (i.e., alternative 2), δ_{alt2}, which is estimated at 0.073 and is only statistically significant at 92% confidence level (robust *t*-ratio of 1.73). The significance of the ZP dummy, δ_{ZP}, reduces from a t-ratio of 5.78 to 3.84, once the cost sensitivity takes on a non-linear form. This implies that ZP effect, albeit still being picked up when cost sensitivity is specified non-linearity, could have been over-stated using a linear-in-cost specification in model SP3-L1 by capturing effects other than the ZP effect.

The comparison of the WTP for the 4G LTE data options between linear and non-linear cost models gives very similar picture as in SP2. WTP values estimated at 3zł and 10zł are significantly lower when cost sensitivity is specified nonlinearly, and vice versa when the 4G LTE package is priced higher at 30zł. In contrast to the model assuming linear cost sensitivity where WTP drops by 22% as mentioned earlier (SP3-L1 vs SP2-L1), differences in WTP computed based on non-linear cost function between SP2 and SP3 are not statistically different across all cost levels, as presented in **Table 4**. The *t*-ratios of the difference in WTP between treatments at 3zł, 10zł and 30zł are 0.90, 0.31 and 0.18, respectively, which are not statistically significant. From a policy perspective, it is clearly more desirable to minimise the bias that is caused by the small cost effect by adopting a flexible functional form for more consistent and robust WTP measures in contrast of the results from using the linear-in-cost specification in our SC data. The insignificant difference in WTP also reassures that the difference in contextual difference of the SC choice format, namely, the removal of the SQ alternative, does not lead to substantial difference in the WTP computation.

Table 4 – WTP differences between SP2 and SP3

Reference cost and data limit		Linear-in-cost model SP3-L1 vs SP2-L1			Non-linear-in-cost model SP3-BC1 vs SP2-BC1		
Data Limit (GB)	Cost (zł)	est.	diff.	rob. *t*-rat (diff)	est.	diff.	rob. *t*-rat (diff)
5	3	2.11	-22%	*2.63*	0.54	48%	*0.90*
5	10	2.11	-22%	*2.63*	1.30	15%	*0.31*
5	30	2.11	-22%	*2.63*	2.94	-8%	*0.18*
20	3	0.53	-22%	*2.63*	0.13	48%	*0.90*
20	10	0.53	-22%	*2.63*	0.33	15%	*0.31*
20	30	0.53	-22%	*2.63*	0.73	-8%	*0.18*

Overall, the choice analysis for SP3 provides evidence of a moderate ZP effect in either cost sensitivity specification. By design, this treatment offers the most robust setting for identifying the ZP effect, without the confounding with the SQ effect, while providing abundant information at cost near zero to improve separation of the ZP effect and non-linearity. We highlight the risk of ZP effect being over-stated if non-linearity in cost sensitivity is not incorporated. Also, presentation of small costs in trade-offs also leads to stronger ZP effect detected than in SP2, for both linear and non-linear cost sensitivity specifications.

4.4 Joint modelling

This paper suggests re-framing the SQ for separating the SQ and ZP effects in SP2 and inclusion of the small costs for distinguishing ZP effect from non-linearity in SP3. This section describes the joint estimation that merges the SC data from all 3 treatments to form a more representative sample for estimation. This allows consolidated platform that allows separation of ZP effect, SQ effect, and non-linearity in utility. Joint estimation can increase robustness for all key parameters including cost coefficients, data limit coefficients and also accessibility for multiple devices that are shared across the 3 treatments. Since respondents are exposed to different choice sets across the 2 treatments, the joint model captures the scale difference by incorporating separate scale parameters for each treatment, denoted as μ_{SP1}, μ_{SP2} and μ_{SP3}, for SP1, SP2 and SP3, respectively. The scale parameter for SP3, μ_{SP3}, is normalised to one. The Parameter estimates and WTP measures from the joint-estimation assuming non-linear cost sensitivity (Model Joint-BC1) are summarised in **Table 5**. This is based on the previous finding from all individual models that non-linear cost sensitivity gives better model fit.

We retain the ZP dummy and the SQ constant, denoted as δ_{ZP} and $\delta_{alt1,SP2}$, respectively, for capturing the ZP and SQ effects in SP2 while the ZP dummy, δ_{ZP}, is also used for capturing the ZP effect in SP3. The confounding ZP and SQ effects are captured solely by an ASC, denoted as $\delta_{alt1,SP1}$. Two ASCs, $\delta_{alt2,SP1/2}$ and $\delta_{alt2,SP3}$, are assigned to capture the presentation order effect for the middle alternatives in SP1 and SP2, and the left-most alternatives in SP, respectively.

Table 5 – Joint estimation results

		Joint - BC1 Box-Cox~(Cost)	
Respondents		302	
Obs		7852	
Final LL		-5982.27	
SP1		-2302.94	
SP2		-2168.74	
SP3		-1510.59	
AIC		11986.5	
Adj. ρ^2		0.190	
Parameter estimates		**est.**	**rob. *t*-rat(0)**
ZP (δ_{ZP})		0.338	4.46
$SQ_{alt1, SP2}$ ($\delta_{alt1, SP2}$)		-0.418	2.40
$ASC_{alt1, SP1}$ ($\delta_{alt1, SP1}$)		-0.341	-1.89
$ASC_{alt2, SP1/2}$ ($\delta_{alt2, SP1/2}$)		0.068	1.73
$ASC_{alt2, SP3}$ ($\delta_{alt2, SP3}$)		0.058	1.39
$Cost_{Box-Cox}$ (β_{cost})		-0.546	-7.51
$Lambda_{Box-Cox}$ (λ)		0.464	9.68
Data $Limit_{log}$ (β_{dlim})		1.231	17.00
Multi-Access (δ_{mdev})		0.386	8.04
$Scale_{SP1}$ (μ_{SP1})		0.852	16.19
$Scale_{SP2}$ (μ_{SP2})		0.828	17.78
$Scale_{SP3}$ (μ_{SP3})		1.000	-
WTP (zł/GB) at reference data limit & cost			
Data Limit (GB)	Cost (zł)	**est.**	**rob. *t*-rat(0)**
5	3	1.035	7.69
5	10	1.700	7.96
5	30	2.673	8.04
20	3	0.259	7.69
20	10	0.425	7.96
20	30	0.668	8.04

The scale parameters for SP1 and SP2, denoted as μ_{sp2} and μ_{sp2}, are estimated to be lower than one, which suggest more variability in choices observed relative to SP3. This is not surprising as respondents are required to handle more alternatives in SP1 and SP2. This is in line with the argument that higher level of task complexity can lead to larger variance in random error term (Swait & Adamowicz, 2001).

The specification of non-linearity in cost sensitivity also leads to decrease in the significance of the ZP effect, from a robust t-ratio of 8.00 (model Joint BC-1) to 4.46 (model Joint LC-1). This is consistent to the earlier findings that ZP effect can be over-estimated when it also captures some of the non-linearities in cost sensitivity due to misspecification. By disentangling the ZP and SQ effects, it can be seen that the SQ constant is estimated at -0.418, with a t-ratio of 2.4. Not only that the ZP effects are disentangled from the SQ effect, but our joint model results based upon non-linear cost formulation indicate that the preferences towards the SQ alternatives are largely due to the ZP effect, while respondents are indeed prefer to trade rather than sticking with free Wi-Fi coverage.

The WTP computed for the joint model largely fall between the values obtained from each treatment and hence is consistent to the previous findings. As shown earlier, the non-linear-in-cost model allows higher cost sensitivity for small cost levels and vice versa for higher costs. This leads to WTP which is 46% lower at small cost level (1.04 vs. 1.93 at 3zł and 5GB; 0.26 vs 0.48 at 20GB) and 39% higher for higher cost level (2.67 vs. 1.93 at 30zł and 5GB; 0.67 vs 0.48 at 20GB) when compared to the linear-in-cost specification. Given that the model has shown better fit by the non-linear specification, it is arguably that ignoring the non-linearity in cost sensitivity could over-state the ZP effect and also lead to over-estimation of WTP at small costs and alternatively under-estimate WTP at high costs in this SC data

5 Conclusions

This paper develops an experimental design that best allows identification of the ZP effect and the separate identification from SQ effects. Our analysis provides evidence of the presence of the ZP effect and suggests that the SQ effect captured in our SC data can largely be explained by the disproportionate attractiveness of the zero cost, rather than the preference of remaining at the SQ. This finding can potentially affect many commonly studied choice situations where ZP alternatives are presented, yet the impacts of capturing the ZP effect on the valuation studies have been under-examined to date.

The experimental approach discussed herein is a relatively straightforward extension of the conventional experimental design in the field of environmental or health economics. This includes modifying the framing of the SQ context to include some non-free SQ options, together with the inclusion of trade-offs at small cost levels. Our analysis is based on results from the 3 treatments that separately test the impacts of these two design features on the discovery of the ZP effect and WTP measures, and also results from a joint estimation that incorporates the preference data from all 3 treatments. For any prospective choice analyses where ZP effect could potentially come into play, both the inclusion of the non-free SQ alternatives and adequate number of trade-offs at small costs could be incorporated in the experimental design for capturing any potential ZP effect to avoid biased parameters and WTP measures. That said, the re-framed context of the SQ is arguably not the SQ anymore, which might lead to the resulting policy measure that is not compatible with the original intent. Considerable effort should be paid to ensure that the SQ context remains largely comparable even when the small out-of-pocket cost is assigned to the SQ and no significant behavioural change is induced as a result of this change of context.

We turn to the implications for WTP calculations. If the ZP effect is real behavioural effect, then not accounting for it in the utility specification would lead to an overestimation of the cost sensitivity and hence under-estimated WTP. By accounting for the ZP effect through the inclusion of a ZP dummy, the ZP effect is effectively separated from the WTP computation. This is supported by our findings that the WTP measures are not affected at all after the ZP effect is split from the SQ effect as both the ZP and SQ effects are separated from the WTP computation. The WTP computed is thus appropriate for policy analysis provided the sole focus is on the marginal rate of substitution between attributes. Without acknowledging the ZP effect, however, analysts could significantly under-estimate the attractiveness of the designed (policy) alternatives when only a slight departure from the zero cost for the SQ alternative may lead to much higher demand for the designed alternatives. Indeed, our model results from the joint model show that the respondents prefer to trade once the SQ is no longer free. This finding from our SC data suggests a significant role of the ZP effect. It is therefore recommended to include the ZP effect in welfare calculation to compensate for the loss of welfare due to simply moving from a free to a non-free option, and conversely for the gain of welfare for moving to a free alternative.

That said, we cannot exclude the possibility that the ZP effect captured could be amplified within a stated choice setting. Namely, the finding of the ZP effect is reinforced by the experimental design which allows more ZP alternatives in choice tasks. Under this circumstances, the ZP effect should be excluded or adjusted accordingly in welfare calculation. The risk of capturing ZP as a survey artefact leads to the recommendation for future research that the ZP effect should also be tested based on the revealed preference data, where the ZP effect can be truly isolated from any survey contextual effect. This is indeed feasible in practice. For instance in transport context, one could observe the change of drivers' choices for switching between a free existing road and a tolled facility to detect the presence of the ZP effect.

With respect to the low WTP observed in the SP3 using the linear-in-cost model (SP3-L1), it can be argued that the model estimates might be biased by the contextual difference by having force choices (Boyle and Özdemir, 2009) with the SQ alternatives removed. Namely, respondents who prefer to remain at SQ would choose the 4G LTE data options with lower cost (and hence higher cost sensitivity and lower WTP) as the SQ is no longer an alternative in SP3. This finding is cross-checked with the models specified with the non-linear cost sensitivity which give better model fit and we found no significant difference in WTP between SP2 and SP3.

This paper also implements alternative ways of accounting for the ZP effect in the utility function and impacts on the WTP measures are empirically tested. Our results suggest that respondents' sensitivities to cost decrease with increasing cost levels. This is supported by the use of Box-Cox transformation of costs that gives significant improvements in model fit. The presence of non-linearities in utility brings complications in the WTP computation. First, small cost levels are required to separate the ZP effect and non-linearity as stated above. We found that the linear-in-cost specification is prone to the small cost effect that WTP would be significantly lowered when more data points allows detection of the higher than average cost sensitivities perceived by respondents at small costs. In contrast, the WTP measures are relatively stable with and without provision of small costs by using a more flexible utility functional form. More importantly, our results show the ZP effect detected in all cases become less significant when non-linear cost sensitivities are specified as opposed to the linear specification which is commonly assumed in many choice analyses. The results suggest that the ZP dummy with the linear-in-cost specification may have captured some of the non-linearities in utility due to utility misspecification. On the other hand, however, if the real source of the retrieved effects is non-linearity in the cost sensitivity rather than a ZP effect, then the inclusion of a ZP dummy with cost sensitivity misspecified linearly may also bias WTP. Our findings strongly support the proposition that flexible utility functions should be tested in capturing any ZP effect.

In conclusion, we demonstrate that capturing the ZP effect requires not only via simple constant term but also a careful design of the experimental design and the appropriate estimation technique to minimise the risk of obtaining biased parameter estimates and WTP measures. Analysts need to tread a fine line between uncovering the full “behavioural effects” and producing results that are useful for policy analysis. Several avenues for further research are identified. These include the testing for the ZP effect in more advanced model structures, such as nesting structures and taste heterogeneity, or a mix of both.

References

Adamowicz, W., Boxall, P., Williams, M. & Louviere, J. 1998. Stated preference approaches for measuring passive use values: choice experiments and contingent valuation. *American journal of agricultural economics,* 80**,** 64-75.

Adamowicz, W., Dupont, D., Krupnick, A. & Zhang, J. 2011. Valuation of cancer and microbial disease risk reductions in municipal drinking water: An analysis of risk context using multiple valuation methods. *Journal of Environmental Economics and Management,* 61**,** 213-226.

Ariely, D. 2008. *Predictably irrational*, HarperCollins New York.

Ben-Akiva, M., McFadden, D. & Train, K. 2019. Foundations of stated preference elicitation: Consumer behavior and choice-based conjoint analysis. *Foundations and Trends® in Econometrics,* 10**,** 1-144.

Box, G. E. & Cox, D. R. 1964. An analysis of transformations. *Journal of the Royal Statistical Society: Series B (Methodological),* 26**,** 211-243.

Boyle, K. J. & Özdemir, S. 2009. Convergent validity of attribute-based, choice questions in stated-preference studies. *Environmental and Resource Economics,* 42**,** 247-264.

Carson, R. T. & Louviere, J. J. 2011. A common nomenclature for stated preference elicitation approaches. *Environmental and Resource Economics,* 49**,** 539-559.

Dallas, S. K. & Morwitz, V. G. 2018. "There Ain't No Such Thing as a Free Lunch": Consumers' Reactions to Pseudo-Free Offers. *Journal of Marketing Research,* 55**,** 900-915.

Daly, A. 2010. Cost damping in travel demand models: Report of a study for the department for transport.

Daly, A., Hess, S. & de Jong, G. 2012. Calculating errors for measures derived from choice modelling estimates. *Transportation Research Part B: Methodological,* 46**,** 333-341.

Douven, R., Van der Heijden, R., McGuire, T. & Schut, F. 2019. Premium levels and demand response in health insurance: relative thinking and zero-price effects. *Journal of Economic Behavior & Organization*.

Ferrini, S. & Scarpa, R. 2007. Designs with a priori information for nonmarket valuation with choice experiments: A Monte Carlo study. *Journal of environmental economics and management,* 53**,** 342-363.

Gaudry, M. J., Jara-Diaz, S. R. & de Dios Ortúzar, J. 1989. Value of time sensitivity to model specification. *Transportation Research Part B: Methodological,* 23**,** 151-158.

Hensher, D. A. 2019. Context dependent process heuristics and choice analysis–A note on two interacting themes linked to behavioural realism. *Transportation Research Part A: Policy and Practice,* 125**,** 119-122.

Hensher, D. A. & Greene, W. H. 2003. The mixed logit model: the state of practice. *Transportation,* 30**,** 133-176.

Hess, S. & Beharry-Borg, N. 2012. Accounting for latent attitudes in willingness-to-pay studies: the case of coastal water quality improvements in Tobago. *Environmental and Resource Economics,* 52**,** 109-131.

Hess, S., Bierlaire, M. & Polak, J. W. 2005. Estimation of value of travel-time savings using mixed logit models. *Transportation Research Part A: Policy and Practice,* 39**,** 221-236.

Hess, S., Daly, A. & Batley, R. 2018. Revisiting consistency with random utility maximisation: theory and implications for practical work. *Theory and Decision,* 84**,** 181-204.

Hess, S., Daly, A., Hensher, D. A. & Adler, T. 2011. The role of constants in WTP computation. *European Transport Conference.* Glasgow, United Kingdom.

Hess, S. & Rose, J. M. 2009. Should reference alternatives in pivot design SC surveys be treated differently? *Environmental and Resource Economics,* 42**,** 297-317.

Hess, S., Rose, J. M. & Hensher, D. A. 2008. Asymmetric preference formation in willingness to pay estimates in discrete choice models. *Transportation Research Part E: Logistics and Transportation Review,* 44**,** 847-863.

Hüttel, B. A., Schumann, J. H., Mende, M., Scott, M. L. & Wagner, C. J. 2018. How consumers assess free E-services: the role of benefit-inflation and cost-deflation effects. *Journal of Service Research,* 21**,** 267-283.
Louviere, J. J., Hensher, D. A. & Swait, J. D. 2000. *Stated choice methods: analysis and applications*, Cambridge university press.
McFadden, D. 1986. The choice theory approach to market research. *Marketing science,* 5**,** 275-297.
McFadden, D., Ben-Akiva, M., Goett, A. & Bolduc, D. 1986. The Choice Theory Approach to Analyze the Preferences and Choices of Electric Utility Consumers. EPRI Final Report RP2671-1.
Meyerhoff, J. & Liebe, U. 2009. Status quo effect in choice experiments: empirical evidence on attitudes and choice task complexity. *Land Economics,* 85**,** 515-528.
Nicolau, J. L. & Sellers, R. 2012. The free breakfast effect: An experimental approach to the zero price model in tourism. *Journal of Travel Research,* 51**,** 243-249.
Oehlmann, M., Meyerhoff, J., Mariel, P. & Weller, P. 2017. Uncovering context-induced status quo effects in choice experiments. *Journal of Environmental Economics and Management,* 81**,** 59-73.
Rich, J. & Mabit, S. L. 2016. Cost damping and functional form in transport models. *Transportation,* 43**,** 889-912.
Samuelson, W. & Zeckhauser, R. 1988. Status quo bias in decision making. *Journal of risk and uncertainty,* 1**,** 7-59.
Scarpa, R., Ferrini, S. & Willis, K. 2005. Performance of error component models for status-quo effects in choice experiments. *Applications of simulation methods in environmental and resource economics.* Springer.
Shampanier, K., Mazar, N. & Ariely, D. 2007. Zero as a special price: The true value of free products. *Marketing science,* 26**,** 742-757.
Train, K. E. 2009. *Discrete choice methods with simulation*, Cambridge university press.
Williams, H. & Ortúzar, J. d. D. 1982. Behavioural theories of dispersion and the mis-specification of travel demand models. *Transportation Research Part B: Methodological,* 16**,** 167-219.
Zhang, J. & Adamowicz, W. L. 2011. Unraveling the choice format effect: A context-dependent random utility model. *Land Economics,* 87**,** 730-743.

Appendix A

Following the notation as in , we define Φ as a differentiable and invertible function of a number of model parameters β. Applying the Delta method, the variance of the function Φ is equal to:

$$var(\Phi) = \sum_{l=1}^{L} \phi_l'^2 \omega_{ll} + 2\sum_{l=2}^{L}\sum_{m=1}^{l-1} \phi_l' \phi_m' \omega_{lm}$$

CASE 1 ($\lambda > 0$): When data limit is log-transformed and cost is Box-Cox transformed, function Φ becomes:

$$\Phi = \frac{\partial V/\partial Dlim_{LN}}{\partial V/\partial Cost_{BC}} = \frac{\beta_{dlim}}{Dlim\, \beta_{cost}(Cost)^{\lambda-1}}$$

Individual elements Φ', which is the first derivative matrix of Φ, and the variance of Φ are given by:

$$\phi_k' = \frac{\partial \Phi}{\partial \beta^*}$$

$$For\ \beta^* = \beta_{dlim}: \qquad \phi_1' = \frac{1}{Dlim\, \beta_{cost}(Cost)^{\lambda-1}}$$

$$For\ \beta^* = \beta_{cost}: \qquad \phi_2' = -\frac{\beta_{dlim}}{Dlim\, \beta_{cost}^2(Cost)^{\lambda-1}}$$

$$For\ \beta^* = \lambda: \qquad \phi_3' = -\frac{\beta_{dlim}(\lambda-1)}{Dlim\, \beta_{cost}Cost^{\lambda}}$$

$$var(\Phi) = \phi_1'^2 \omega_{11} + \phi_2'^2 \omega_{22} + \phi_3'^2 \omega_{33} + 2(\phi_2'\phi_1'\omega_{21} + \phi_3'\phi_1'\omega_{31} + \phi_3'\phi_2'\omega_{32})$$

CASE 2 ($\lambda = 0$): When data limit is log-transformed and cost is log-transformed, then Φ, Φ_k', and $var(\Phi)$ are given by:

$$\Phi = \frac{\partial V/\partial Dlim_{LN}}{\partial V/\partial Cost_{LN}} = \frac{\beta_{dlim}Cost}{\beta_{cost}Dlim}$$

$$For\ \beta^* = \beta_{dlim}: \qquad \phi_1' = \frac{Cost}{\beta_{cost}Dlim}$$

$$For\ \beta^* = \beta_{cost}: \qquad \phi_2' = -\frac{\beta_{dlim}Cost}{\beta_{cost}^2 Dlim}$$

$$var(\Phi) = \phi_1'^2 \omega_{11} + \phi_2'^2 \omega_{22} + 2\phi_2'\phi_1'\omega_{21}$$

CASE 3: When data limit is log-transformed, while cost is linear, Φ, Φ_k', and $var(\Phi)$ become:

$$\Phi = \frac{\partial V/\partial Dlim_{LN}}{\partial V/\partial Cost_{linear}} = \frac{\beta_{dlim}}{\beta_{cost}Dlim}$$

$$For\ \beta^* = \beta_{dlim}: \qquad \phi_1' = \frac{1}{\beta_{cost}Dlim}$$

$$For\ \beta^* = \beta_{cost}: \qquad \phi_2' = -\frac{\beta_{dlim}}{\beta_{cost}^2 Dlim}$$

$$var(\Phi) = \phi_1'^2 \omega_{11} + \phi_2'^2 \omega_{22} + 2\phi_2'\phi_1'\omega_{21}$$