Quantum rotation: a new method for capturing a change of perspective

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ABSTRACT

Quantum probability, first developed in theoretical physics, has recently been used to model previously unexplainable data in cognitive psychology. This has led to the recent development of choice models based on quantum probability. This paper tests whether these new models can also capture 'changing states' or equivalently 'changing perspectives' in moral choice contexts. We apply models with a quantum logic framework to two distinctly different choice case-studies. In the first one, respondents have to make choices between 'sacred' and a 'secular' attributes of route alternatives ('a taboo trade-off'). The second study investigates how an individual weighs wage and commuting times for themselves relative to the wages and commuting times for their partners. For the first scenario, we find that 'quantum rotations' can accurately capture the addition of a taboo trade-off. For the second, quantum rotations can explain the difference in making trade-offs affecting just the decision-maker compared to trade-offs involving both the decision-maker and their partner.

Keywords: Quantum probability, route choice, moral choice, travel behaviour

1. INTRODUCTION

Moral choice situations can be summarised as those where a decision maker feels that the choice alternatives can to some extent be categorised as 'right' or 'wrong'. As a result, the associated choices can be perhaps more complex as they do not involve straightforward trade-offs between attributes of alternatives. For example, a decision-maker may not choose what they want to choose as they believe it to be an immoral option. While moral choice behaviour has received much attention in economics and psychology, it is rarely considered in the choice modelling literature [Chorus, 2015]. This is despite the fact that many typical experiments conducted for understanding moral preferences use paradigms such as variations of the infamous trolley problem (where a 'runaway trolley' has two possible paths, both of which will result in the death of some individual(s), and the decision-maker must choose who to save), for which a precise understanding of the trade-offs that are being made could be obtained using choice models. This is perhaps due to the fact that moral preferences are difficult to investigate outside of the laboratory, with typical experimental methods for examining moral choice scenarios often suffering from low external validity [Bauman et al., 2014]. However, more recently, moral choice behaviour has become more prominent to the travel behaviour modelling community through, for example, the reinvention of the trolley problem as a self-driving car problem [Awad et al., 2018]. Thus far, there has not been much consideration given to the types of choice models used for the modelling of such scenarios, despite the wide range of theoretical explanations for moral behaviour that have been proposed [Chorus, 2015]. However, some steps towards the development of choice models specifically for moral choice contexts have been made [Chorus et al., 2018].

In this paper, we specifically look at models based on quantum logic. These have not yet been applied to moral choice scenarios, despite the adoption of such methods 'allowing for a re-examination of the challenge of formalising psychological concepts of conflict, ambiguity, and uncertainty' [Wang et al., 2013]. Quantum logic, originally developed to help explain ordering effects in physics [Birkhoff and Von Neumann, 1936], has recently made a significant impact in cognitive psychology [Bruza et al., 2015]. A key difference between classical and quantum probability is that under quantum theory, the distributivity law of probability ($A(B+C) = AB+AC$) fails to hold [Hancock et al., 2019]. Crucially, this means that the adoption of quantum logic allows for an elegant and convenient framework for understanding many 'paradoxical' findings which become 'intuitive' [Wang et al., 2013], such as probability judgement errors [Busemeyer et al., 2011], question ordering effects [Trueblood and Busemeyer, 2011] and violations of the 'sure thing principle' [Pothos and Busemeyer, 2009][1]. With, for example, ordering effects also frequently observed in choice modelling applications, it is unsurprising that quantum models have also since made the transition into choice modelling [Lipovetsky, 2018]. Furthermore, quantum models can be used to accurately capture the 'change of decision context and mental state' when moving between choices made under revealed preference and stated preference settings [Yu and Jayakrishnan, 2018].

Additionally, it has been demonstrated that quantum logic can be implemented into choice models to accurately understand route choice problems as well as best-worst choice behaviour

[1] A classic example of a probability judgement error is given by Tversky and Kahneman [1983], who found that participants, after reading 'Linda was a philosophy major. She is bright and concerned with issues of discrimination and social justice', were more likely to agree with the statement 'Linda is a feminist bank teller' than the statement 'Linda is a bank teller'. The 'sure thing principle' states than an individual who would take the same action if some event happens or not should also take that action without knowing the outcome of the event [Savage, 1954].

in the context of route choice [Hancock et al., 2019]. Thus there appears to be ample scope for further developments of quantum choice models, with our previous development of the notion of a 'quantum rotation' within a choice model providing useful transitions across choice contexts. The aim of this paper is thus to test whether similar rotations can also be used to accurately capture changes in choice context within moral choice scenarios. We apply the models to two very different datasets. The first allows us to test whether quantum rotations can be used to capture the impact of the presence of a 'taboo trade-off' [Chorus et al., 2018] involving trade-offs between 'sacred' and 'secular' goods. The second tests whether quantum rotations can be used to capture differences between how an individual weighs wage and commuting times for themselves relative to the wages and commuting times for their partner.

The remainder of this paper is organised as follows. First, we give a description of how quantum logic works. Next, we detail how it has provided useful explanations for choices with a moral component in cognitive psychology. We then detail how we mathematically build a quantum choice model, before applying it to our two moral choice datasets. We finish with some conclusions and directions for future research.

2. QUANTUM LOGIC

In this section, we first give a general overview of how quantum logic works. Next, we demonstrate how it can be used to capture a change in perspective. We then demonstrate how it has been used to explain a number of 'paradoxical' phenomena in cognitive psychology, many of which have moral components. Finally, we demonstrate how we mathematically operationalise quantum logic into the models utilised in this paper.

2.1. How quantum logic works

Under quantum models, each choice scenario is represented by a multidimensional 'Hilbert' space, which is spanned by a set of orthonormal vectors, with one vector for each possible alternative. For a basic example of this, we refer to a previous example given in the introduction of Hancock et al. [2019], where an individual is choosing whether to commute to work by car or by train (See Figure 1).

The preference of an individual is initially represented by a unit 'state' vector which is denoted $|z\rangle$. The action of making a choice is represented by a projection (ρT or ρC) from the state vector onto the vector representing the chosen alternative ($|T\rangle$ or $|C\rangle$ in Figure 1). The projections are not represented by the arrows from the initial state, but by the lines from the origin in the same direction as the chosen alternative, represented by $|\rho T|$ and $|\rho C|$. Crucially, this means that we can set the probability of an alternative being chosen as the squared length of this projection. As the state vector is of unit length and the different alternatives are represented by a set of orthonormal vectors, the set of squared length projections must sum to one by Pythagoras' Theorem.

2.2. Capturing a change in perspective

If two choices are equivalent, then they can be represented by the same vectors within a Hilbert space. However, 'incompatible' choices are represented by different vectors. For example, if a decision-maker has to chose their favourite and least favourite alternative from a set, the sensitivities for what constitutes the best alternative may not be equivalent to what constitutes the worst. This can be represented in quantum models through different vectors for an alternative being the best compared to the same alternative being the worst. To capture the change of perspective (con-

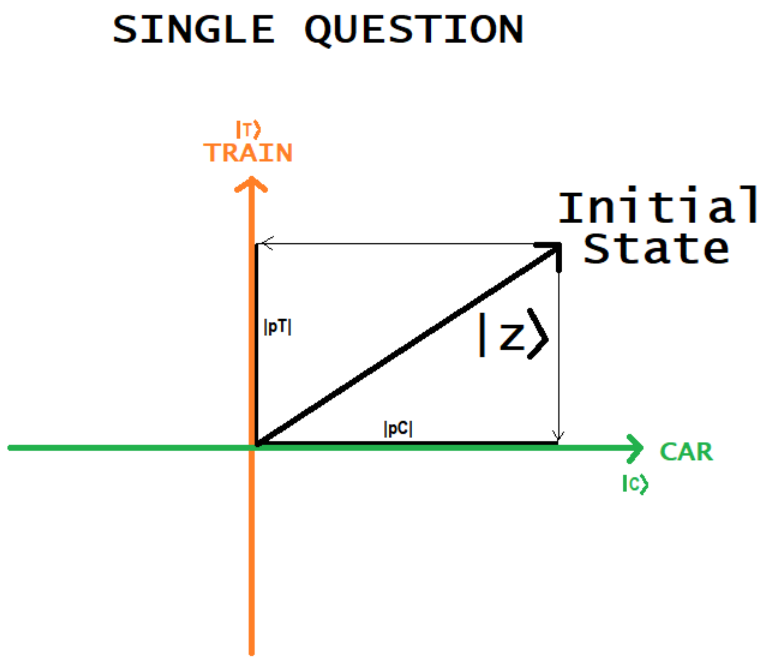

FIGURE 1 : A basic question under quantum probability

sidering the best, to considering the worst), a 'quantum rotation' is required, which maps the state vector from the basis of vectors representing the choice of alternatives as being the best, to the basis of vectors representing the choice of alternatives as being the worst. It has previously demonstrated that this rotation accurately captures the difference between best and worst choice [Hancock et al., 2019]. In this paper, we use the same concept of quantum rotation to capture changes of context in moral choice scenarios. For example, we can consider how an individual might evaluate their mode choice differently if they were first asked 'how environmentally friendly are you?' The addition of this moral component may result in the decision-maker considering their alternatives from a different perspective. This is the case only if the two choices (yes or no to being environmentally friendly and car or train) are incompatible. However, as all four combinations of answers to these two questions are clearly possible, these choices cannot be represented by the same set of basis vectors, thus new vectors for environmentally friendly and environmentally unfriendly (see Figure 2) are required.

The result of this is that when a decision-maker makes one choice, their state (under a quantum model) updates. If, for example, they decide that they are environmentally friendly, their state vector would then be represented by the environmentally friendly vector itself. This results in a change in the lengths of projection onto the vectors representing the choice of car or train. Thus, in this example, the individual becomes more likely to choose to travel by train if they first decide that they are environmentally friendly. The result of this is that quantum models can capture a change in perspective through a quantum rotation, which can be mathematically represented simply by estimating the impact a change of basis has on the lengths of projections (see the later section of

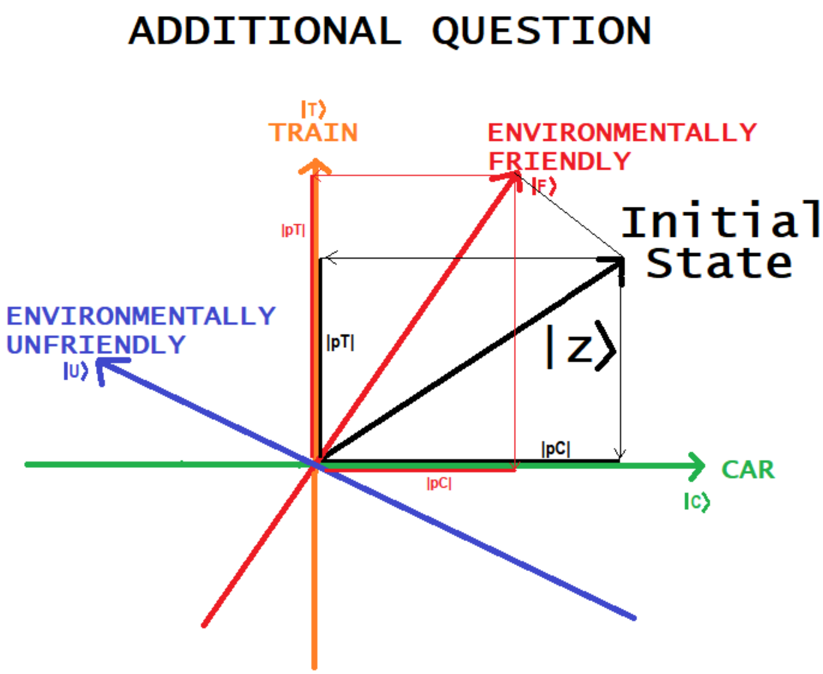

FIGURE 2 : The impact of an additional moral component on mode choice under a quantum model

this paper for the mathematics of this).

2.3. Quantum theory and moral psychology

Whilst choice models with a quantum logic framework have not yet been tested on moral choice data, there have been a number of applications of quantum logic to experimental results with a moral component in cognitive psychology. In particular, quantum logic has been used to explain ‘interference’ effects where an additional question impacts the probability of an action. For example, Busemeyer et al. [2009] tested the impact of additionally asking decision-makers to categorise a face as ‘good’ or ‘bad’ before choosing how to respond (‘withdraw’ or ‘attack’), finding that quantum logic could be used to accurately capture the difference in responses with and without the categorisation task. Furthermore, in jury decision-making experiments, where participants read strong or weak defences and prosecutions, quantum logic provided a better account of the ordering effects that were observed compared to models based on classical probability [Trueblood and Busemeyer, 2010]. Ordering effects observed when participants state opinions about political figures can also be explained by quantum cogntition [Pothos and Busemeyer, 2013]. In the context of a ‘taboo trade-off’, where an individual can sacrifice ‘sacred’ goods in favour of ‘secular’ goods, a similar interference may take place in that a decision-maker may not wish to appear ‘unethical’. Similarly, an individual may consider their own happiness differently if they are also required to consider the happiness of their partner. For this reason we use quantum models to test for interference effects in both of the choice datasets that we test in this paper.

2.4. Mathematical outline for a model based on quantum logic

Whilst quantum logic provides a convenient structure for capturing phenomena in cognitive psychology, the operationalisation of quantum logic into a choice model is less simple. The key component (as discussed in detail by Hancock et al. [2019]) is that the state vector is of unit length and the different alternatives are represented by a set of orthonormal vectors. This means that the complete set of squared projection lengths must sum to one. Consequently, we can build quantum choice models by developing methods for defining a state vector. In this process, negative values should be avoided, given that squaring is symmetrical about zero. The state vector additionally should be based on functions of the attributes of the alternatives.

For the applications in this paper, we consider two alternative approaches. The first of these is based on 'Quantum Pairwise Comparison Framework A', as defined by Hancock et al. [2019]. The key feature of this model is that the use of regret-like functions [Chorus, 2010] allows for a state vector (which can be represented by a series of projection lengths) to be created with strictly positive values. For the applications in this paper, we simplify this definition by not additionally considering weights for the relative importance of the comparison for a particular pair of attributes. The length of projection for an alternative i for individual n in choice task t is thus defined as:

$$|\rho_{int}| = \delta_{QPCA,i} + \sum_{k=1}^{K} \sum_{j \neq i} ln(1 + e^{\beta_k (x_{intk} - x_{jntk})}), \tag{1}$$

where $k = 1, ..., K$ is an index across attributes, $\delta_{QPCA,i}$ is an alternative specific constant[2], β_k are attribute-specific weights and j is the other alternative.

The second approach considered in this paper is a model based on sine and cosine functions [cf. Lipovetsky, 2018]. Whilst models based on these functions will likely not work for three or more alternatives (see the example in Hancock et al. [2019] for an explanation of this), we only have two attribute levels in the first dataset in this paper (which additionally makes the use of the second model developed by Hancock et al. [2019] less desirable). For a model based on sines and cosines, the length of projection for the first alternative can be defined as:

$$|\rho_{nt}| = sin\left(\delta + \sum_{k=1}^{K} \beta_k \cdot (x_{intk} - x_{jntk})\right), \tag{2}$$

where δ is a constant and β_k is a set of attribute-specific weights as before. The length of projection for the second alternative then simply uses a cosine rather than a sine function.

For the quantum rotations applied in this paper, we only need to estimate a change of basis matrix, M^* (of size 2×2 or 3×3 for the respective datasets in this paper), which will appropriately adjust the length of the projections for the different alternatives depending on the impact that the change of perspective has on the choice being made. The result of this is that the new projections, ρ^*, can be defined based on the change of basis matrix M^* and the previous projections ρ. For the binary choice case, this is defined as:

$$|\rho^*| = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \times |\rho|, \tag{3}$$

[2]Note that given the probabilities with which the different alternatives are chosen are based on squaring these projection lengths, the same probabilities will be generated if all lengths are multiplied by the same factor (as oppose to the addition of the same factor in random utility models). Thus the number of constants that are identifiable is equal to the number of alternatives).

with the extension to a trinary choice scenario simply involving a matrix of size 3×3 instead.

3. EMPIRICAL APPLICATION

In this section, we first detail the two datasets that are used for testing our quantum rotation models. We then apply both models with and without quantum rotations, demonstrating how they can capture changes in perspective in moral contexts.

3.1. Datasets

The first dataset we use involves 'taboo trade-offs' [Chorus et al., 2018]. Decision-makers choose between the introduction of a new transport policy or keeping the status quo. To simplify the choice scenarios, each new policy offered simply an increase or decrease for four attributes: 300 EUR vehicle ownership tax, 20 minutes travel time for each car commuter per day, 100 serious injuries in traffic accidents and 5 deaths in traffic accidents. This results in a total of 16 possible new policies, which are offered in turn to each of 99 decision-makers, resulting in a dataset with a total of 1,584 choices. Chorus et al. [2018] then define a choice as involving a 'taboo trade-off' if a decision-maker could choose a policy that involves decreasing tax or travel time (a secular attribute) at the cost of increasing the number of injuries or deaths (a sacred attribute).

The second dataset we test involves decision-makers completing two sets of choice tasks based on an individual's willingness to accept longer commutes for better salaries [Beck and Hess, 2016]. The first set involved trade-offs between the individual's current travel time and salary or an increased salary (of 500 or 1000 SEK in net wage per month) at a cost of an increase in one-way travel time (of either 10 or 25 minutes). The second set additionally included attributes for increased travel time and salaries for the partner of the decision-maker, meaning that the decision-maker has to make choices about who to prioritise. While the first may involve typical time-cost trade-offs that can potentially be captured well with RUM models, the latter involves a more complex decision context without any 'crisp' trade-off element in that there may not be a clear ethical protocol for how to make the decision. All choice tasks included a status quo alternative, a new alternative and a 'I am indifferent' option. A sample of of 1,179 households (with both partners in each household, resulting in 2,358 individuals) completed 4 tasks for the first set involving only attributes affecting themselves, and 4 or 5 tasks for the second set with attributes impacting both members of the household. This resulted in a total of 20,041 choice observations.

3.2. A quantum rotation for taboo trade-offs

For the first set of models tested, we do not include any parameters to control for the presence of a taboo trade-off. We test a logit model as a comparison against both of the two quantum models specified by Equations 1 and 2 respectively. The results of these models are given by Table 1. For both the quantum pairwise comparison model (QPCA) and the trigonometric quantum model (TQ), we find that the impact of the additional alternative specific constant is insignificant, thus we have 5 parameters for each of the models. For all of the models in this paper, we use R packages maxLik [Henningsen and Toomet, 2011] and apollo [Hess and Palma, 2019] for estimation of the log-likelihood functions. In this case, neither basic quantum model performs as well as the basic logit model. Whilst QPCA appears to find nearly identical ratios for the estimates for the different attribute coefficients, the trigonometric quantum model assigns a higher importance to travel time.

Next, we apply quantum rotations. In this case, a quantum rotation is used to capture the shift in perception of the alternatives in the presence of a taboo trade-off. Thus, if the decision-maker

TABLE 1 : Results of basic models applied to the taboo trade-off dataset

Model	Pars.	LL	BIC	Ratios		
				Tax/Time	Injuries/Time	Deaths/Time
Logit	5	-721.23	1,479.30	1.88	2.14	1.52
QPCA	5	-725.40	1,487.64	1.90	2.13	1.49
TQ	5	-722.94	1,482.72	1.68	1.85	1.32

can decrease travel time or tax at the cost of increasing the number of fatalities or serious injuries, a quantum rotation is applied to the estimated projection lengths. As multiplying each projection by the same factor will result in no change in the probabilities with which the alternatives are chosen, the first element of the rotation matrix must be fixed to one. We then test three different rotations with varying amounts of flexibility: a diagonal matrix (QR1), a symmetrical matrix (QR2) and a fully flexible matrix (QR3). The results of these models are given in Table 2, with the rotation matrix elements that are estimated given in bold.

TABLE 2 : Results of the quantum rotation models applied to the taboo trade-off dataset

Quantum Pairwise Comparison A				Quantum rotation matrix			
Version	Parameters	Log-likelihood	BIC	M[1,1]	M[1,2]	M[2,1]	M[2,2]
Basic	5	-725.40	1,487.64	1.00	0.00	0.00	1.00
QR1	6	-718.78	1,481.77	1.00	0.00	0.00	**1.40**
QR2	7	-716.90	1,485.37	1.00	0.18	**0.18**	**1.18**
QR3	8	-716.49	1,491.92	1.00	**0.32**	**-0.06**	**1.55**
Trigonometric Quantum				Quantum rotation matrix			
Version	Parameters	Log-likelihood	BIC	M[1,1]	M[1,2]	M[2,1]	M[2,2]
Basic	5	-722.94	1,482.72	1.00	0.00	0.00	1.00
QR1	6	-717.33	1,478.86	1.00	0.00	0.00	**1.29**
QR2	7	-714.21	1,479.99	1.00	0.12	**0.12**	**1.32**
QR3	8	-713.63	1,486.20	1.00	**0.30**	**-0.17**	**1.84**

For both types of quantum models, we see a significant improvement in log-likelihood from the addition of 1 rotation parameter. However, whilst the addition of further parameters continues to improves log-likelihood, this results in a worse BIC. Crucially, the ‘Generic Taboo Trade-Off Aversion’ (TTOA) model (which is a logit model with an additional parameter that adds a penalty for the presence of a taboo trade-off) achieves a log-likelihood of -719.47 and a BIC of $1,483.15$ [Chorus et al., 2018], values which are both worse than the QR1 models tested here. The trigonometric quantum model outperforms the quantum pairwise comparison model, as a contrast to previous results [Hancock et al., 2019]. This is in part possibly due to the fact that there is less variation in the attribute levels in the choice tasks. Notably, all rotation models have a value greater than 1 for element $M_{2,2}$, which, all else being equal, would mean that the decision-maker is more likely to choose the 2nd alternative (which is the status quo alternative) in the presence of a taboo trade-off (in line with the results of Chorus et al. 2018).

TABLE 3 : Impact on probabilities as a result of quantum rotations

Scenario	Attributes				Taboo Trade-Off?	Share of support					
	Tax	Time	Injuries	Deaths		Observed	TTOA	QPCA QR3 before	QPCA QR3 after	TQ QR3 before	TQ QR3 after
1	-	-	-	-	No	98.0%	92.0%	96.9%		97.4%	
2	-	-	-	+	Yes	68.7%	71.0%	74.9%	66.4%	77.5%	67.0%
3	-	-	+	+	Yes	29.3%	21.0%	19.2%	21.8%	31.7%	24.6%
4	-	+	+	+	Yes	11.1%	9.0%	5.1%	11.3%	11.3%	12.1%
5	+	+	+	+	No	2.0%	1.0%	0.6%		2.1%	
6	+	-	-	-	No	62.6%	63.0%	64.9%		63.4%	
7	+	+	-	-	No	44.4%	37.0%	37.7%		38.1%	
8	+	+	+	-	No	4.0%	6.0%	2.3%		3.4%	
9	-	+	-	+	Yes	42.4%	46.0%	48.8%	42.7%	53.6%	41.0%
10	+	-	+	-	Yes	15.2%	15.0%	12.2%	16.8%	18.3%	16.3%
11	-	-	+	-	Yes	46.5%	56.0%	58.6%	50.7%	64.4%	51.3%
12	-	+	-	-	No	80.8%	81.0%	85.6%		83.7%	
13	-	+	+	-	Yes	30.3%	31.0%	31.5%	30.2%	39.1%	29.7%
14	+	-	-	+	Yes	22.2%	26.0%	24.2%	25.2%	30.7%	24.0%
15	+	-	+	+	Yes	5.1%	4.0%	0.4%	5.6%	1.2%	4.8%
16	+	+	-	+	No	7.1%	11.0%	7.5%		10.6%	
Mean absolute deviation from true share of support (percentages; all choice tasks)							3.03	2.41		2.01	
Mean absolute deviation from true share of support (percentages; taboo tasks only)							2.68	2.21		1.93	

However, the different estimates for $M_{1,2}$ and $M_{2,1}$ mean that the overall impact is more complex for QR2 and QR3. Table 3 gives the probability of supporting the new policy before the quantum rotation and after it for both QR3 models. Crucially, in both models, the predicted shares of supporting a policy are closer on average to the observed shares than in the TTOA model.

For QPCA, the probabilities tend to become less extreme, whereas they often become more extreme under the TQ model. Crucially, however for both models, if the lengths are similar, then the probability for the status quo increaseses in the presence of a taboo trade-off. Additionally, both models have smaller mean absolute deviations from the true share of support than the TTOA model.

3.3. A quantum rotation when you consider your partner

For the datasets on willingness to accept longer commutes for better salaries, there are two distinct choice sets: the first only includes factors impacting the decision-maker, the second includes impacts on the partner. Consequently, we first test the difference between models that treat the two sets as the same (and thus have the same parameter estimates for the alternative specific constants and utility coefficients for the impact of changes in salary and travel time for the decision-maker, across both choice sets). The second set of models is essentially made up of two separate components, as it has entirely different sets of parameters for the different choice sets. We test both random regret minimisation models (which were previously demonstrated [Hess et al., 2014] to be effective for this dataset due to the presence of an indifferent option) as well as the quantum pairwise comparison model based on Equation 3. All models simply estimate a constant for the regret (for RRM) or projection length (for the quantum model) for the indifferent alternative. The results of these models are given in Table 4.

TABLE 4 : Models with and without separate sets of parameters for the two different choice sets

Model	Separate Pars.	Free Parameters	Log-likelihood	BIC
RRM	No	6	-12,784.21	25,628
RRM	Yes	10	-12,426.71	24,952
QPCA	No	7	-12,624.13	25,318
QPCA	Yes	12	-12,289.38	24,698

Regardless of whether RRM and QPCA are compared with or without separate parameters, the results indicate that the quantum models provide a large gain in model fit as well as substantially lower BIC values. Additionally, both QPCA and RRM find clear evidence that separate sets of parameters can be used to improve model fit, demonstrating that there is an inconsistency in how a decision-maker considers factors impacting themselves compared to when there are also factors impacting their partner.

Crucially, however, this inconsistency or ‘change of mindset’ incurred through changing from thinking about just yourself compared to yourself *and your partner* could be captured by a quantum rotation. Thus, for our quantum rotation models, we estimate a single set of coefficients that apply to choices made in both choice sets, and instead include a quantum rotation matrix for adjusting the projection lengths appropriately when additionally considering travel time and salary changes for the partner. We again test diagonal, symmetric and fully flexible matrices. The results of these models are given in Table 5.

TABLE 5 : Results from quantum rotations when considering travel time and salary changes for your partner

#	Model	Parameters	Log-likelihood	BIC	Log-likelihood Gain	
					Over basic model	As % of Model 2
1	Basic	7	-12,624.13	25,318	-	-
2	Separate	12	-12,289.38	24,698	334.75	100%
3	QR1	9	-12,436.03	24,961	188.10	56%
4	QR2	12	-12,332.37	24,784	291.76	87%
5	QR3	15	-12,278.06	24,705	346.07	103%

Whilst additional flexibility for the quantum rotation matrices did not significantly improve model fit for the taboo trade-off dataset, we see a consistent improvement both in log-likelihood and BIC for the quantum rotation models here. Additionally, in line with previous results, quantum rotation models can again provide better model fit than a model that allows for a separate set of parameter values [Hancock et al., 2019], although in this case the BIC is slightly better for a model with separate parameters. This suggests that these models provide a suitable framework for capturing changes in choice context.

For the most complex of these models, the rotation matrix estimated is:

$$M = \begin{bmatrix} 1.000 & -0.464 & 4.834 \\ -0.046 & 0.116 & 2.631 \\ 0.151 & -0.068 & -0.651 \end{bmatrix}, \tag{4}$$

where the first alternative is to stay with the status quo, the second is to increase travel times and salaries and the third is the indifferent option. The high values for $M_{1,3}$ and $M_{2,3}$ indicate that if an individual is indifferent for a choice scenario involving just changes for themself, then they will likely not still be indifferent if there are additionally changes for their partner.

4. CONCLUSIONS

Moral decision making is becoming an interesting area of application for choice modelling, calling for the development of appropriate model specifications. Given the recent successful transition of quantum logic into cognitive psychology, it is unsurprising that the results in this paper suggest that quantum logic could also have a significant impact in choice modelling. In particular, the results in this paper demonstrate that ‘quantum rotations’ accurately capture a change in decision context when a moral element enters the dimension of choice.

Whilst the results here are positive, it is not clear that they are distinctly *better* than those of Hancock et al. [2019]. This implies that we cannot necessarily attribute the success of the quantum models in this paper to the fact that there are moral components in the choices modelled. This is particularly clear from the result that our quantum model already has better model fit than the random regret model for the 2nd dataset tested in this paper *before* the moral component was captured through a quantum rotation. Further tests of quantum logic based models could further enlighten whether they are models that are particularly suited to moral decision-making, or whether they are suitable for decision-making in general.

Additionally, further models could consider different sorts of moral choice data. For example, quantum models may be well suited for modelling choices made in 'moral machine' choice tasks. Results may also differ significantly for revealed preference datasets, with the concern of external validity of stated moral choices [Bauman et al., 2014] still an issue that has yet to have been addressed.

Overall, however, our results indicate that choice models with a quantum logic framework have vast potential, both within moral choice scenarios and more generally.

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