

INCORPORATING RESPONSE TIME IN A DECISION FIELD THEORY MODEL

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1 ABSTRACT

2 Decision field theory (DFT), popular in mathematical psychology, has recently been used in choice
3 modelling for consumer choices and route choices. One of the key differences that DFT has
4 from typical choice models is that it has preference values for each alternative that update over
5 time. This results in a different probability of picking each alternative depending on how long
6 a decision-maker considers their alternatives. Despite this psychologically feasible assumption,
7 computational complexities of calculating the probability of alternatives in a DFT model have re-
8 sulted in the number of deliberation timesteps often being set to a high value, thus ignoring and
9 failing to utilise the dynamic nature of DFT. However, recent advances in the underlying computa-
10 tional methods for DFT have allowed for the calculation of alternatives at any time point. Thus, we
11 build on this work by allowing the estimate for the number of deliberation timesteps to vary as a
12 function of the response time of a decision-maker. We demonstrate that DFT model fit can be im-
13 proved by considering response times using both route choice and conservation programme choice
14 datasets. Consequentially, the dynamic nature of decision field theory could allow it to become an
15 important model for revealed preference datasets such as gap acceptance in driving behaviour.

1 INTRODUCTION

2 Decision field theory, first developed in the 1990s (*1*) is a dynamic, stochastic choice model that
3 updates over time. It considers preference values for each alternative that update with each timestep
4 as the decision-maker considers the different attributes of the alternatives. The decision-maker
5 then comes to a conclusion either when an alternative reaches some satisfactory threshold value
6 or when the decision-maker runs out of time to make the decision. For example, a driver might
7 reach traffic lights at which point they are forced to choose which direction to turn next, even if
8 they are not satisfied with one of the alternatives. Decision field theory has been used extensively
9 in mathematical psychology but has only recently made the transition into choice modelling (*2, 3*).
10 It was initially used as a model for understanding risky choice decisions with two alternatives with
11 two attributes but has since been expanded to allow for multiple attributes and multiple alternatives
12 (*4*). It has been used widely across the mathematical psychology literature, having been used to
13 model a variety of choices including decision-making in sport (*5*), monetary gambles (*6*), likely
14 crime suspects (*7*) and consumer decisions (*8*). There has thus far been limited use of DFT in
15 transport, with a key difficulty having been the calculation of the probability of alternatives. The
16 computationally heavy simulations required have resulted in many applications in mathematical
17 psychology not using DFT's dynamic nature by fixing the number of deliberation timesteps to a
18 high value (*7, 9*). Arguments have been made, however, that DFT models perform better when
19 a low number of timesteps are used (*10*). Thus, there has been a requirement to improve the
20 computational methods behind DFT such that the number of timesteps does not need to be set to a
21 high value.

22 One of the first comparisons of DFT against traditional choice models such as multinomial
23 logit found that for most participants, DFT fit the data best (*11*). This application, however, only
24 considered two alternatives, for which probabilities of alternatives in a DFT model do not require
25 a number of timesteps to be estimated. The first comparison of DFT against traditional choice
26 models for multi-alternative choice also found that DFT performed comparably to multinomial
27 logit, and also probit (*2*). Again, the number of timesteps was not estimated, as the probability
28 of alternatives were only calculated after the preference values stabilised (which happens under
29 certain assumptions within a DFT model, demonstrated later in this paper). The present paper
30 builds on the latest application of DFT in choice modelling (*3*), where computational developments
31 demonstrated that the probability of alternatives can be calculated at any moment in a DFT model.

32 Under a decision field theory model, a decision-maker can come to a conclusion and stop
33 deliberating when they reach some internal threshold for one of the alternatives (equivalent to
34 satisficing (*12, 13*), where a participant chooses one of the alternatives if it is 'good enough').
35 Alternatively, they might at some point run out of time (for example, reaching the end of an access
36 road and having to choose whether to force their way onto a motorway or wait to be let in by
37 another driver), at which point the alternative with the highest preference value is chosen.

38 Whilst it would be impossible to measure abstract internal preference thresholds, which are
39 set arbitrarily or for convenience in many applications of DFT (*4, 10, 14*), and are likely to vary
40 amongst decision-makers, it is possible to know the point at which a decision-maker makes their
41 decision. Thus, if we can record how long a decision-maker takes to make a decision, then re-
42 gardless of whether the decision-maker stopped due to an internal or external stopping point, we
43 know that at the moment they made the decision, under DFT, the alternative they chose had the
44 highest preference value. Whilst we cannot know how many iterations of preference value updat-
45 ing occurs each second, it is possible to add this in as a function of the response time. In this way,

1 combined with avoiding the sacrifice of time being set to infinity, we can restore DFT to being a
 2 properly dynamic model in which the time taken to make a decision impacts the probability of each
 3 option being selected. This has already been demonstrated to be very effective for understanding
 4 participants' (single) travel mode choice (15).

5 There have been some studies considering response time in choice modelling, with systematic
 6 differences found depending on the size of the difference between travel times and costs in a choice
 7 task compared to those from a reference trip. (16). There have also been suggestions that response
 8 times reflect how much cognitive effort a participant uses (17), in which case longer response times
 9 would suggest more deterministic behaviour.

10 This paper demonstrates for the first time how the decision-maker's response time can be natu-
 11 rally incorporated into a DFT model such that the probability of alternatives across multiple choices
 12 is impacted. The remainder of this paper is organised as follows. First, we present the methodology
 13 behind decision field theory, demonstrating how the probability of alternatives are calculated and
 14 how this is impacted by the number of deliberation timesteps. Next, we demonstrate how response
 15 time can be incorporated into decision field theory models for two typical stated choice datasets.
 16 Finally, we conclude by drawing some conclusions and presenting directions for future research.

17 **METHODOLOGY**

18 In this section we describe how the probability of alternatives are calculated under a decision field
 19 theory model and demonstrate how reparameterising the timestep parameter results in a natural
 20 method for response time to be incorporated into the model.

21 **Basic theory**

22 Decision field theory is a dynamic model, meaning that preferences for alternatives accumulate
 23 and de-accumulate over time. At each timestep, the preference values are updated as follows:

$$P_t = S \cdot P_{t-1} + V_t \quad (1)$$

24 The previous values are stored in a column vector, P_{t-1} , and are multiplied by a feedback matrix,
 25 S , after which a valence vector V_t is added. The feedback matrix has two parameters that allow for
 26 the attraction, similarity and compromise effects to occur (4) and is defined:

$$S = I - \phi_2 \times \exp(-\phi_1 \times D^2) \quad (2)$$

27 where ϕ_1 is a memory parameter and ϕ_2 is a sensitivity parameter. D is the distance between the
 28 attributes of the alternatives. D can include a factor to control for the importance of the different
 29 attributes (10), but the Euclidean distance between the attributes can also be used to avoid an
 30 additional parameter (15). The memory parameter allows for preferences to naturally increase or
 31 decline over time. If this value is less than one, preference values stabilise for a large number of
 32 timesteps (2, 4). The sensitivity parameter affects how much the alternatives compete with each
 33 other, and allows for more similar alternatives to deduct higher amounts of preference from each
 34 other. This is in effect a similar concept to a nested logit model, although tests have not yet been
 35 carried out to test whether DFT models capture nesting effects as robustly.

36 At each timestep, a decision field theory model assumes that the decision-maker compares a
 37 single attribute across all of the alternatives. For example, a decision-maker might attend to the
 38 costs of the alternatives at the first time point. This results in a random valence vector at time t , V_t ,

1 which can be calculated:

$$V_t = C \cdot M \cdot W_t + \varepsilon_t \quad (3)$$

2 where C is a contrast matrix used to rescale the values such that they total zero, M is the matrix of
 3 attribute values and $W_t = [0..1..0]'$ with entry $j = 1$ if and only if attribute j is the attribute being
 4 attended to by the decision-maker at timestep t . This means that a DFT model estimates $n - 1$
 5 weights (where n is the number of attributes), where each weight, w_j corresponds to the proportion
 6 of time the decision-maker considers attribute j . The total attention time sums to one, therefore
 7 the final attribute weight can be calculated once the others are known. As $\sum_j w_j = 1$, a standard
 8 uniform distribution $X \sim U(0, 1)$ can be used to select which attribute a decision-maker attends to
 9 at each time step. There is also a random error vector, $\varepsilon_t = [\varepsilon..\varepsilon]'$, added on to allow for noise
 10 and variation in the probability values that DFT predicts. Higher values of ε would be expected
 11 for more complex decision-making tasks (10). Whilst different values could be estimated for the
 12 different alternatives in ε_t , we use one value for all alternatives in this paper.

13 Estimation of the probability of alternatives in a decision field theory model

14 The probabilities of alternatives at time t can be calculated once the expected value and the covari-
 15 ance of the preference values (ξ_t and Ω_t) are known (4).

16 To calculate the expected value of the preference values, we first expand equation 1, which
 17 results in:

$$P_t = \sum_{k=0}^{t-1} S^k \cdot V_{t-k} + S^t \cdot P_0 \quad (4)$$

18 where P_0 is the initial preference vector. The attribute weights w_j are stationary, therefore W_t can
 19 be considered a stationary stochastic process. This results in V_t also being a stationary stochastic
 20 process with mean $E[V_t]$ and a variance covariance matrix given by $Cov[V_t]$. We let ε_t vary accord-
 21 ing to a normal distribution with mean zero and variance ε . Thus if we have $\mu = E[V_t]$ then it can
 22 be calculated as $\mu = C \cdot M \cdot w_m$, where w_m is a vector containing the probabilities of each of the
 23 attributes being considered. We also have $Cov[V_t] = \Phi = C \cdot M \cdot \Psi \cdot M' \cdot C' + \varepsilon$, where $\Psi = Cov[W_t]$
 24 (C and M are matrices of constants). We can then calculate the expected value and the expected
 25 covariance of P_t . With S being a constant, $E[P_t]$ reduces to:

$$E[P_t] = \xi_t = \sum_{k=0}^{t-1} S^k \cdot \mu + S^t \cdot P_0 \quad (5a)$$

$$= (I - S)^{-1}(I - S^t) \cdot \mu + S^t \cdot P_0 \quad (5b)$$

26 We can also now calculate the covariance of the preference values:

$$Cov[P_t] = \Omega_t = Cov \left[\sum_{k=0}^{t-1} S^k \cdot V_{t-k} + S^t \cdot P_0 \right] \quad (6a)$$

$$= \sum_{k=0}^{t-1} \left[S^k \cdot \Phi \cdot S^{k'} \right] \quad (6b)$$

27 To avoid the computationally-heavy summation required to calculate this covariance, a simpli-
 28 fication can be made by assuming that $t \rightarrow \infty$ (2). If the eigenvalues of the feedback matrix, S , are
 29 less than 1, then $S^t \rightarrow 0$ and hence the expected values is reduced to

$$\xi_{\infty} = (I - S)^{-1} \cdot \mu \quad (7)$$

1 Moreover, the computationally intensive summation is also avoided such that

$$\overline{\Omega}_{\infty} = (I - Z)^{-1} \cdot \overline{\Phi} \quad (8)$$

2 Where the feedback matrices are replaced with a matrix Z of size $n^2 \times n^2$ and Φ (with entries
3 p_{ij}) is reshaped into a column matrix:

$$Z = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1n^2} \\ z_{21} & z_{22} & \dots & z_{2n^2} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n^2 1} & z_{n^2 2} & \dots & z_{n^2 n^2} \end{bmatrix} \quad \overline{\Phi} = \begin{bmatrix} p_{11} \\ p_{21} \\ \vdots \\ p_{n1} \\ p_{21} \\ \vdots \\ p_{nn} \end{bmatrix} \quad (9)$$

4 However, if the response time taken by the decision-maker is to be used, then we must further
5 simplify $Cov[P_t]$ such that we avoid requiring a summation whilst simultaneously keeping the
6 timestep parameter. By setting $z_{(j-1)n+i, (k-1)n+l} = S_{il} S'_{kj}$, $Cov[P_t]$ reduces (3):

$$Cov[P_t] = \Omega_t = \sum_{k=0}^{t-1} [S^k \cdot \Phi \cdot S^{k'}] \quad (10a)$$

$$= \sum_{k=0}^{t-1} [Z^k \cdot \overline{\Phi}] \quad (10b)$$

$$= (I - Z)^{-1} (I - Z^t) \overline{\Phi} \quad (10c)$$

7 This simplified form for Ω_t , together with ξ_t mean that we can now calculate the probabilities
8 of the alternatives at any moment t . On the basis of the multivariate central limit theorem, P_t
9 converges to the multivariate normal distribution (4). The chosen alternative is the alternative with
10 the greatest preference value at the conclusion of the deliberation process. Thus the probability of
11 choosing alternative A from a set of n alternatives at time t is:

$$Prob \left[\max_{i \in n} P_t[i] = P_t[A] \right] = \int_{X>0} exp \left[-(X - \Gamma)' \Lambda^{-1} (X - \Gamma) / 2 \right] / (2\pi |\Lambda|^{0.5}) dX \quad (11)$$

12 with $X = [P_t[A] - P_t[B], \dots, P_t[A] - P_t[n]]'$, $\Gamma = L\xi_t$, $\Lambda = L\Omega_t L'$ where

$$L = \begin{bmatrix} 1 & -1 & 0 & \dots & \dots & 0 \\ 1 & 0 & -1 & \ddots & & \vdots \\ 1 & \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & \vdots & & \ddots & -1 & 0 \\ 1 & 0 & \dots & \dots & 0 & -1 \end{bmatrix} \quad (12)$$

13 L is a matrix comprised of a column vector of 1s and a negative identity matrix of size $n - 1$ where
14 n is the number of attributes. The column vector of 1s is placed in the i^{th} column where i is the
15 chosen option.

1 **The impact of time in decision field theory**

2 Under a decision field theory model, the preference of alternatives is impacted by the number of
 3 timesteps for which the decision-maker considers the choices available. We can also see from
 4 figure 1 how the probability of alternatives would change under the parameter values shown as the
 5 number of timesteps increases.

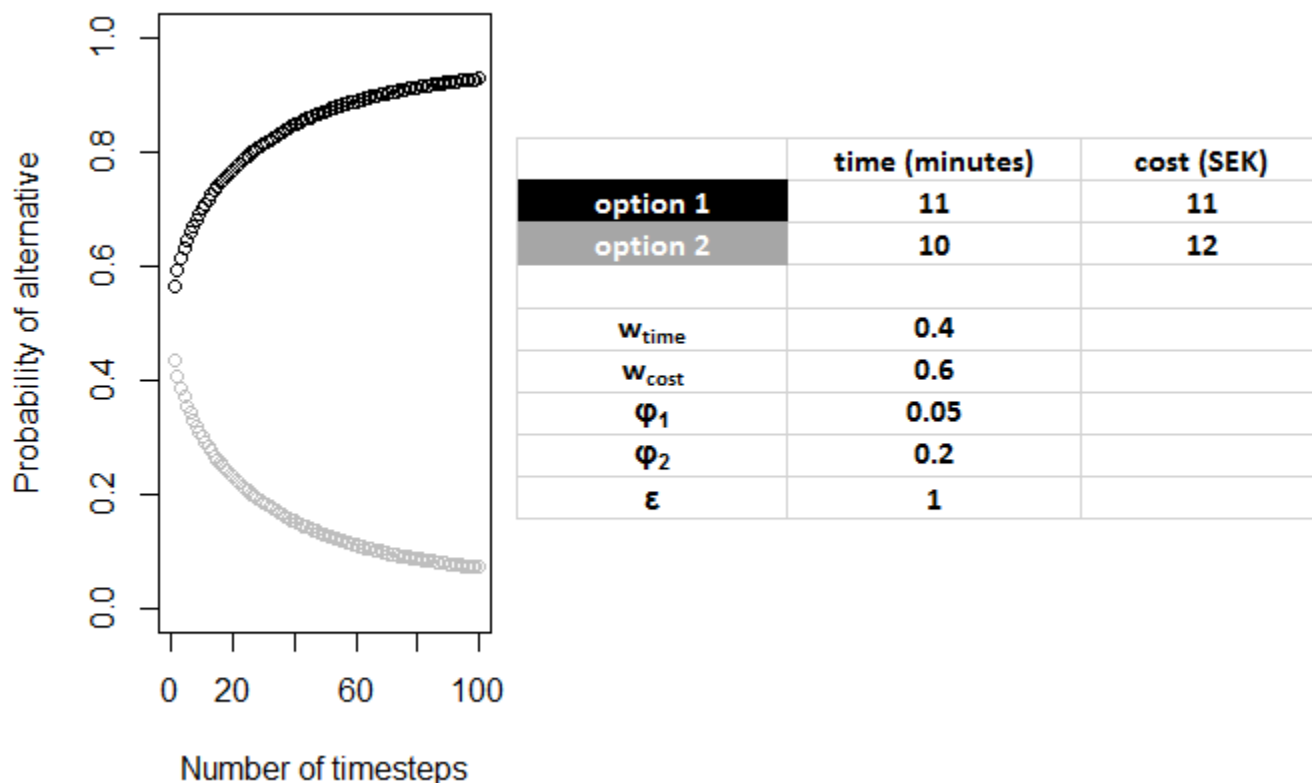


FIGURE 1 : The probability of two alternatives as the number of timesteps increases

6 At first, the impact of the weights is minimal, such that the chosen alternative depends on which
 7 attribute is considered first. As the number of timesteps increases, the higher weight for cost begins
 8 to have an impact, with the cheaper alternative gradually becoming more likely to be chosen. As
 9 ϕ_2 is less than zero, the preference values stabilise for a large number of timesteps (2), and hence
 10 the probability of choosing the alternatives stabilises also. Thus, for example, if a decision-maker
 11 considered an alternative for ten seconds at a rate of three preference updates per second, they
 12 would consider the alternatives for thirty timesteps. In general, the functional form for the number
 13 of timesteps could include multiple parameters:

$$timesteps = f(rt) + g(I) \tag{13}$$

14 where f is a function of the response time, rt , and g is a function of the attributes of a person, I .
 15 For example, elderly individuals may not process information as quickly, hence they may require a
 16 larger number of timesteps before coming to the same conclusions. Whilst previous applications of

1 decision field theory have simply estimated or fixed the number of timesteps, this paper considers
 2 a number of methods for incorporating response time into the timestep parameter. We look at the
 3 impact of including linear as well as log terms for the relationship between the response time and
 4 the number of timesteps. Several applications of accumulation models in mathematical psychology
 5 have already considered the prediction of response times (18, 19), therefore it is also possible that
 6 a latent structure could be considered, such that latent variables predict both the response time
 7 and the number of timesteps used to make a decision. This would also possibly be a method for
 8 avoiding potential measurement error. This is, however, beyond the scope of the current paper.

9 EMPIRICAL APPLICATIONS

10 We will now demonstrate how the response time can be used for two typical stated choice datasets.

11 Route choice

12 The route choice dataset tested in this paper comes from a study on response time patterns in a
 13 stated choice experiment (16). In each choice task, respondents have two alternatives described by
 14 travel time and travel cost (in Swedish Krona). The study included respondents who completed
 15 the survey on the internet and some who completed it on the telephone. As the response times
 16 in the telephone sample include the time taken for the choice task to be read out, they are not
 17 directly comparable, therefore were omitted. Furthermore, we also discard choices with a recorded
 18 response time of 0 seconds and those with a response time of more than 60 seconds, as we assume
 19 that the respondent was either not attempting to respond to the choice seriously or was interrupted.
 20 Finally, we omit any choices made by respondents who have less than six (out of eight) choice
 21 tasks remaining. This leaves us with 15,546 choice tasks completed by 2,358 respondents. We
 22 apply two multinomial logit models to the dataset. The more complex has five parameters and
 23 defines the utility for alternative j as:

$$U_j = ASC_j + \beta_{cost} * cost + \beta_{time} * time + \beta_{lcost} * \log(cost) + \beta_{ltime} * \log(time) \quad (14)$$

24 The simpler model with only three parameters does not include terms for the log of cost and the
 25 log of time. The results of the model are given in table 1. To check if incorporating the response
 26 time has an impact on a DFT model, we first fit a DFT model without including response time,
 27 simply estimating the number of timesteps to be some constant, t .

TABLE 1 : Model results for the route choice dataset

model	par.	LL	AIC	BIC
MNL-1	3	-8,769.75	17,545	17,552
MNL-2	5	-7,753.52	15,517	15,528
DFT	5	-6,909.67	13,829	13,840
DFT-T1	7	-6,908.74	13,831	13,847
DFT-T2	8	-6,899.82	13,816	13,833
DFT-T3	5	-6,900.67	13,811	13,822

28 The results from table 1 demonstrate that, despite having the same number of parameters, DFT
 29 comprehensively outperforms MNL in terms of model fit. This table also shows the results from
 30 our models incorporating response time. The first specification (DFT-T1) for including response
 31 time sets the number of timesteps as:

$$timesteps = t + \tau_1 * rt + \tau_2 * \log(rt) \quad (15)$$

1 where rt is the response time and t is a constant. However, it appears that this does not improve
 2 model fit and table 2 suggests that neither of our τ terms was significant. We therefore adjust the
 3 second model such that it additionally attempts to incorporate differences across participants by
 4 considering a term τ_3 for capturing the impact of a participant's mean response time. DFT-T2
 5 therefore has the specification:

$$timesteps = t + \tau_1 * rt + \tau_2 * \log(rt) + \tau_3 * mrt \quad (16)$$

6 where mrt is the mean response time for the individual. This does significantly improve model fit.
 7 Furthermore, the third and final specification (DFT-T3), which only uses the mean response time:

$$timesteps = \tau_3 * mrt \quad (17)$$

8 achieves a similar log-likelihood whilst simultaneously having the same number of parameters
 9 as the DFT model without response time. It therefore appears that for this dataset, whilst incor-
 10 porating response time per choice task has little impact, incorporating the average response time
 11 of respondents does significantly improve the model fit. Considering figure 1, this implies that
 12 respondents who take longer on average make more deterministic choices.

TABLE 2 : Parameter estimates for the decision field theory models for the route choice dataset

model	DFT		DFT-T1		DFT-T2		DFT-T3	
	est.	rob. t-rat.	est.	rob. t-rat.	est.	rob. t-rat.	est.	rob. t-rat.
LL	-6,908.92		-6,908.74		-6,899.82		-6,900.67	
w_{TC}	0.5961	81.16	0.5961	112.17	0.5961	111.76	0.5957	84.72
ϕ_1	0.0004	3.64	0.0004	3.54	0.0004	3.31	0.0004	3.45
ϕ_2	0.5756	14.50	0.5741	15.02	0.5703	15.15	0.5712	14.69
ϵ	1.2074	0.53	1.1088	0.74	1.5020	0.84	1.0261	0.52
t	4.5447	13.77	5.0903	9.69	4.3424	5.06	-	-
τ_1	-	-	0.0218	0.88	0.0431	1.33	-	-
τ_2	-	-	-0.3551	-0.99	-1.1403	-2.11	-	-
τ_3	-	-	-	-	0.1707	3.54	1.7337	15.91

13 Conservation choice

14 The conservation dataset tested in this paper comes from a study exploring tree planting prefer-
 15 ences in a stated preference survey (20). Whilst not strictly a transport application, this survey used
 16 a very typical stated preference format for conducting the survey, with choice tasks setup in the
 17 same format as they are in many transport applications. It was also part of a broader project explor-
 18 ing carbon emissions and global warming, issues also important in transport choice modelling. 146
 19 participants completed 16 stated choice tasks where they were asked which of two conservation
 20 programmes they preferred. Each of the programmes was described by four attributes: country
 21 (Senegal or Peru), provision of online (OI) information (Yes/No), type of programme (restorative
 22 or preservative) and cost (2,5,10,15 EUR). In all tasks, the participant also had a third 'status quo'

1 alternative where they choose not to invest in either of the presented programmes. We exclude
 2 choices with a recorded response time of less than a second or choices with a time of more than
 3 a minute. This leaves us with 2,334 (out of 2,336) choice tasks. We compare the performance of
 4 DFT to a multinomial logit model where the utility of programme j is calculated as

$$U_j = ASC_j + \beta_{OI} * OI + \beta_{TYPE} * TYPE + \beta_{PLACE} * PLACE + \beta_{COST} * COST \quad (18)$$

5 The utility of the status quo option is simply estimated as a constant, ASC_{SQ} . The results of the
 6 MNL models are given in table 3.

TABLE 3 : Results and parameter estimates from models for the conservation dataset

	MNL with 6 parameters		MNL with 4 parameters	
LL	-1,956.537		-1,958.683	
AIC	3,911.07		3,911.37	
	est.	rob. t-rat.	est.	rob. t-rat.
ASC_1	0.0992	2.37	-0.0931	2.28
ASC_{SQ}	-2.3699	-12.12	-2.3739	-12.14
β_{OI}	0.5341	9.24	0.5325	9.27
β_{TYPE}	0.0524	1.11	-	-
β_{PLACE}	0.0445	1.25	-	-
β_{COST}	-0.1370	-10.60	-0.1371	-10.57
	DFT without response time		DFT with response time	
LL	-1,952.596		-1,935.031	
AIC	3,913.19		3,884.06	
	est.	rob. t-rat.	est.	rob. t-rat.
w_{SQ}	0.6183	22.66	0.6408	35.39
w_{OI}	0.1601	6.26	0.1405	10.87
w_{COST}	0.1798	8.12	0.1970	16.59
w_{ASC_1}	0.0418		0.0217	
ϕ_1	0.0132	1.29	3.3729	17.07
ϕ_2	0.8200	106.42	0.0710	3.85
ε	3.4956	7.10	4.5364	3.21
t	17.9668	62.88	-	-
τ	-	-	4.2479	5.03

7 Whilst the coefficients for the type and country of the programmes are insignificant, the MNL
 8 model with more parameters achieves a slightly better AIC value. However, to allow for DFT to be
 9 more competitive, we exclude these parameters in the DFT model, as it already has an additional
 10 three parameters. There are multiple methods for incorporating parameters equivalent to alternative
 11 specific constants in a DFT model (3). In this case, we use an additional weight for time spent
 12 considering the status quo (w_{SQ}) and a weight to capture the underlying bias in picking the first
 13 option (w_{ASC_1}). This fourth weight is fixed as one minus the sum of the other weights. This results
 14 in seven parameters in the DFT model without response times, for which the number of timesteps,
 15 t , is estimated. For the DFT model incorporating response time, we use the same set of parameters

1 but instead set the number of timesteps as linear function of the response time:

$$timesteps = \tau \cdot rt \quad (19)$$

2 where rt is the response time in seconds and τ is a parameter to be estimated. From table 3 we
3 can see that by including the response time, DFT makes a significant gain in log-likelihood. This
4 results in achieving a lower AIC value compared to MNL, whereas before it was higher.

5 CONCLUSIONS

6 The work in this paper was motivated by the recent improvements in the computational mecha-
7 nisms underlying decision field theory (3). With it now being easily possible to incorporate de-
8 cision response time into a DFT model, this paper looked at the impact of this on two datasets.
9 For our conservation dataset, directly setting the number of timesteps as a linear function of the
10 response time resulted in improved model fit. Whilst this was not the case for our route choice
11 dataset, we did find that including a decision-maker's mean response time improved model fit.
12 Both results suggest that a decision-maker makes more deterministic choices if their response time
13 is longer, as previously implied (17).

14 The fact that response time can be directly included in calculating the probability of alternatives
15 may have some impact in stated preference studies but is far more likely to have an impact in
16 work involving revealed preference data, where decisions such as when to merge lanes are more
17 likely to be impacted by time pressure. Additionally, previous work has demonstrated that the
18 linear ballistic accumulator model (19), a rival dynamic choice model also from mathematical
19 psychology, can be adjusted such that changing information can be incorporated into the model.
20 Similarly, it is possible that as well as incorporating response time, a decision field theory model
21 could incorporate changing attributes, such as the speed at which the car in front is moving in a
22 car-following model. This could also prove useful for studying how commuters change their route
23 choice when forced to do so due to a change to their original schedule, such as a delayed train. In
24 particular, dynamic models such as DFT may prove useful for forecasting, particularly if we do
25 not have much information on the decision-maker but there is some indication on how long they
26 might take to make the decision.

27 Further work should consider latent constructs, where latent variables are used to predict both
28 response times and the number of timesteps. Certain individuals may process information at dif-
29 ferent rates, meaning that there is likely to be variation in the estimated number of timesteps per
30 second. It may not be that individuals who spend longer considering a decision are necessarily
31 considering the alternative in more detail, as implied by the structure we impose on the number of
32 timesteps in this application. Using random parameters to capture this difference across individuals
33 therefore may have much more explanatory power than when they are fixed.

34 We therefore conclude that, where possible, response times in choice decisions should be
35 recorded, as it is likely that dynamic models such as DFT would be able to use this information
36 to better predict the choices made. This may lead to more accurate estimations and forecasts in
37 many situations, including but not limited to driving behaviour, route choice and also longer-term
38 choices such as whether to buy season tickets or own a car.

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