Accommodating correlation across days in multiple-discrete continuous models for activity scheduling: estimation and forecasting considerations

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Abstract

The MDCEV modelling framework has established itself as a preferred method for modelling time allocation, with data very often collected through travel or activity diaries. However, standard implementations fail to recognise the fact that many of these datasets contain information on multiple days for the same individual, with possible substitution between days. This paper discusses how the theoretical accommodation of these effects is not straightforward, especially with budget constraints at the day and multi-day level. We instead rely on additive utility functions where we accommodate correlation between activities at the within-day and between-day level using a mixed MDCEV model, with multi-variate random distributions. We put forward adaptations of the standard Pinjari and Bhat (2010) forecasting approach to allow us to make links across days also in model application. Finally, we illustrate the issue and the methods using two different time use datasets, confirming our theoretical points and highlighting the benefits of allowing for correlation across days in estimation and substitution in forecasting.

Keywords: MDCEV; activity modelling; forecasting; multi-day time use

1 Introduction

Understanding and modelling the way in which individuals allocate time across different activities is a key topic in travel behaviour research. The data for such research often come from travel or activity diaries, where increasingly, there has been a move away from paper based logbooks to smartphone (or other GPS logger) based digital surveys.

Over the course of a day, an individual allocates time to a set of different activities, with the amounts of time differing across those, where individuals can also decide to not

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engage in a particular activity on a given day. The choice process is thus one of choosing amongst different activities, that are no longer mutually exclusive (as they would be in a discrete choice context) and determining a continuous time allocation for each.

Starting from the late 1950s, econometric models accommodating both a discrete and a continuous dimension of choice were developed (e.g. De Jong, 1990; Dubin and McFadden, 1984; Heckman, 1977; Tobin, 1958; Train, 1986). Nowadays, the state-of-the-art model for accommodating both a discrete and a continuous element of choice is the Multiple Discrete-Continuous Extreme Value (MDCEV) model, especially with the work of Bhat (2008). This model is a generalisation of a multinomial logit model (MNL) for multiple discrete continuous choice contexts and has a simple closed-form probability which is practical even with a large number of alternatives. The model is based on the Kuhn Tucker (KT) first-order conditions for constrained random utility maximisation, previously employed by Hanemann (1978) and Wales and Woodland (1983), which are used to derive the optimal consumption for the given random utility specification subject to a linear budget constraint. The MDCEV model has become a popular tool for modelling time allocation (Kapur and Bhat, 2007; Wang and Li, 2011), where the specification of a time budget is conceptually easier than a money budget, albeit there is scope for doing both at the same time.

A key recognition in the time use survey and modelling literature is that the way an individual allocates time may be poorly understood by a single day snapshot, and numerous studies thus rely on multi-day surveys of varying length, for different research purposes, and using different methods (Arentze and Timmermans, 2009; Chow and Nurumbetova, 2015; Jara-Díaz et al., 2008; Kang and Scott, 2010). Some of these studies have accommodated interactions between different days, such as substitutions in time use between weekdays and weekend days (Bhat and Misra, 1999; Yamamoto and Kitamura, 1999). Minnen et al. (2015) argues that multi-day data allow to observe temporal regularities and habitual behaviour. Jara-Díaz and Rosales-Salas (2015), in their analysis of the duration of time diary data, recommend the collection of a week of data when the aim is to model time allocation, or at a minimum two or three days with appropriate weighting.

While multi-day data has the potential to lead to important insights into behaviour, it also creates further complexity in modelling. Modelling each individual day on its own by assuming separate 24 hour budgets remains behaviourally reasonable, as the decision about the activities to conduct (say during a week) is likely made day by day and not in one go, at the beginning of the week. Estimating day-specific model coefficients also allows an analyst to capture differences across days, but at the same time this approach ignores the fact that "links" between days may exist. While a number of previous applications of MDCEV have made use of such multi-day data (Chikaraishi et al., 2010; Nurul Habib et al., 2008; Spissu et al., 2009), it is not always clear what interactions, if any, between days have been accommodated. These "links" can represent correlations or complementarities across days. It is for example likely that, in a household, the person who performs certain household obligations on one day is more likely to perform them on the other days (an example of positive correlation caused by common unobserved heterogeneity), or maybe the parent who does the drop-off of the children on one day will not do it on the next (negative correlation), or that a person who performs out-of home social activities on one day also performs travel (complementarity).

On the other side, an overall model assuming (for example) a weekly budget of 168 hours, while creating a link between different days, would not allow us to understand differences in the utilities of activities across days (see Calastri et al., 2017, for an application using two days). This can be understood by noting that the estimation of day-specific coefficients would also require day-specific budgets. By not imposing the latter, the analyst would neglect an essential constraint which is present in the data on which the model is based. The issue becomes even clearer when thinking about forecasting. In the first case, i.e. with day specific budgets, a change in the utility of an activity on one day would have no effect on the time spent on any activities on other days, as there is no link between days. This is quite unlikely in reality, as, for example, if an individual is unable to work on a certain day, he/she will try to make up for it by working a little longer on the previous/following day. In the second case, i.e. using an overall budget, substitution between days becomes possible, but the model would not be constrained to allocating only 24 hours across all activities on a given day. Any forecasts would thus not allow us to understand what happens on individual days, but only at the aggregate level. These considerations highlight the fact that it is important to research solutions that allow the introduction of correlations as well as complementarities across different days in MDCEV models of time use, while avoiding violations of the 24 hour budget constraint so as to be able to obtain consistent forecasts.

In this paper, we start by recognising that some activities are more similar than others, i.e. that there is correlation between days as well as across days. An analysis seeking to capture these "links" needs to recognise that these can include correlations due to common unobserved heterogeneity, substitution, or complementarity. When a model is estimated with only one such effect, for example correlations, then these could be due to common heterogeneity or substitution (if the correlation is negative) or complementarity (if the correlation is positive). It is usually not easy to disentangle the different sources of correlation, but the analyst can make an informed guess on the primary source of correlation. We investigate possible theoretical solutions, i.e. ways to develop a model framework that allows to incorporate correlations and complementarities in the MDCEV model across different days. We first show that this problem cannot be solved without making major changes to the model structure. To deal with these issues in a way that is computationally tractable, we put forward a mixed MDCEV model which directly accommodates correlations between activities within and across different days. We then discuss how the presence of inter-day correlations leads to a need for changes to the standard approach used for forecasting with MDCEV models, and we propose two different departures from that framework.

The remainder of this paper is organised as follows. Section 2 discusses the limitations of the *"standard"* implementation of MDCEV models for multi-day data, highlights the complexity of working with non-additive utility functions and puts forward a mixed MDCEV solution within an additive framework. Section 3 discusses the use of this models in forecasting and shows the required changes to the standard forecasting approach. We illustrate the modelling and forecasting approaches using two different datasets in Section 4. Finally, Section 5 summarises our findings and presents directions for future work.

2 Methodological considerations

In this section, we first discuss the limitations of the standard framework, before briefly looking at the use of a non-additive utility specification. We next put forward a mixed MDCEV model as a solution to the problem of working with multi-day data. Finally, we contrast these two solutions.

2.1 Base specification

The random utility specification of the MDCEV model, as introduced by Bhat (2008) is given by:

$$U(x) = \sum_k \frac{\gamma_k}{\alpha_k}[exp(\beta' z_k + \epsilon_k)] \left(\left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \tag{1}$$

so that $U(x)$ is quasi-concave, increasing and continuously differentiable with respect to the vector of consumptions x_k, and $\psi_k = [exp(\beta' z_k + \epsilon_k)]$. ψ_k is the baseline utility of good k, i.e. the marginal utility of the good at zero consumption. It is a function of observed characteristics of the decision maker and of good k, z_k, which also includes a constant representing the generic preference for good k. In this random specification, a multiplicative i.i.d. log-extreme value error term is introduced in the baseline utility.

The analyst can solve the optimal expenditure allocation (with respect to the money spent on goods $1...K$):

$$Max\ U(e_1 \dots e_K) \quad s.t. \sum_{k=1}^{K} e_k^* = E \tag{2}$$

where e_k^* are the optimal amounts of expenditure on goods $1...K$, that exhaust the budget E, and where $e_k = x_k p_k$. This problem is solved by forming the Lagrangian and applying the KT conditions, as detailed in Bhat (2008). The resulting model probability of the expenditure pattern where M goods are chosen, results in the closed-form expression below:

$$
\begin{aligned}
&P(e_1{}^*, e_2{}^*, \ldots, e_M{}^*, 0, \ldots, 0) \\
=&\frac{1}{\sigma^{M-1}}\left(\prod_{i=1}^{M} c_i\right)\left(\sum_{i=1}^{M}\frac{1}{c_i}\right)\left(\frac{\prod_{i=1}^{M} e^{V_i/\sigma}}{\left(\sum_{k=1}^{K} e^{V_k/\sigma}\right)^M}\right)(M-1)! \qquad (3)
\end{aligned}
$$

where σ is a scale parameter (not estimated in our case, as there is no price variation across products), $c_i = \frac{1-\alpha_i}{e_i{}^* + \gamma_i p_i}$ and $V_k = \beta' z_k + (\alpha_k - 1)\ln\left(\frac{e_k{}^*}{\gamma_k p_k} + 1\right) - \ln p_k$ $(k = 1, 2, 3, \ldots, K)$. As explained in Bhat (2008), equation 3 can also be expressed in terms of consumption quantities. In our case, the two forms are interchangeable because there is no price variation, and in the remainder of our discussion we will refer to consumption quantities (x_i).

To model multi-day time-use choices, it is desirable to formulate a unified multiple discrete-continuous choice model that simultaneously recognises the day-level (24 hour) constraints individuals face, with as many constraints as the number of days modelled, and the interactions between activity time allocations across different days, such as substitution and complementarity across different days. This is because ignoring the day-level constraints while accommodating interactions across days can potentially lead to time allocations that either exceed or fall short of 24 hours on a day. On the other hand, a model without interactions across days does not allow for exogenous changes on a day to influence time allocations on another day.

2.2 Multi-day utility maximisation with day-level time constraints and non-additive utility functions

A first specification to model multi-day time use while accommodating day-level time constraints and interactions in time-use across different days is to use non-additive utility formulations that allow for explicit interactions between utility functions of different days while allowing for day-level constraints. One way to formulate non-additive utility functions is to begin with an additive utility form discussed in the previous section and add multiplicative utility terms that interact pairs of utility terms of activity participation on different days, as in Bhat et al. (2015). The parameters estimated on such multiplicative utility terms capture substitution and complementarity between the two choice alternatives being interacted. Specifically, $U = \sum_{l=1}^{L}\sum_{k=1}^{K} u_{kl} + \sum_{k=1}^{K}\sum_{q,m=2,q\neq m}^{L} \theta_{kqm}[u_{kq} \times u_{km}]$, where θ_{kqm} are the parameters estimated on the multiplicative utility terms $[u_{kq} \times u_{km}]$, interacting utility derived from activity k on two different days q and m, to capture substitution and complementarity patterns between the two days.

2.3 Multi-day utility maximisation with day-level time constraints and correlated, additive utility functions

We will now look at our proposed use of a mixed MDCEV model to capture intra and inter-day correlations.

Let us consider the situation in which we have data from N separate individuals, where L_n days are observed for person n. On each day, individual n allocates time to K different activities ($k = 1, 2, \dots, K$), where $K = 1$ is an activity that is performed by every person on every day (i.e. an outside good). In this specification, it is assumed that an individual makes his/her time use choices across different days to maximise the total utility derived from time allocation choices on all days under consideration; subject to as many day-level constraints as the number of days, where the time allocation to all activities on each day sums up to 24 hours. Specifically, $U = \sum_{l=1}^{L} \sum_{k=1}^{K} u_{kl}$ is maximised subject to L day-level time budget constraints $\sum_{k=1}^{K} x_{kl} = 24, \forall l = 1, 2, \dots, L$.

In this specification, U is the total multi-day utility derived by the person, $u_{1l} = \psi_{1l} x_{1l}^{\alpha}$ is the utility from time allocation x_{1l} to the outside good on day l ($l = 1, 2, \dots, L$), $u_{kl} = \frac{\psi_{kl}\gamma_{kl}}{\alpha}\left(\frac{x_{kl}}{\gamma_{kl}} + 1\right)^{\alpha} \forall k = 2, \dots, K$ is the utility from time allocation x_{kl} to activity k on day l, ψ_{kl} is the corresponding baseline utility parameter, γ_{kl} is the corresponding translation parameter, which allows for corner solutions as well as having a role in relation to satiation (with higher γ_{kl} implying lower satiation for activity k and day l). Finally, α is the generic satiation parameter. Note that the subscript for person n is suppressed for ease in notation.

To introduce interactions across activities within a day and across different days, we introduce correlations through common mixing distributions in the utility functions, in both the baseline utility parameters as well as the translation parameters. As can be observed from the utility function, the utility derived from each activity on each day is assumed to be additively separable from that of other such utilities, and specified using a standard MDCEV utility function.

We remain with the example of L_n days for person n, with K different activities per day. We further assume that there are D different types of days, where a simple approach would use the same specification for a given activity on each day, while at the extreme end, we would have 7 different treatments. In our exposition below, we generally focus on the situation where weekdays (WD) are treated differently from Saturdays (SAT) and Sundays (SUN).

A number of different specifications are possible with the MDCEV model, where our empirical work uses a generic α parameter across activities, which we (after testing) set to 0, along with activity specific γ parameters for the $K-1$ activities that are not treated as outside goods. In our theoretical discussions, we ignore the possibility of including socio-demographic effects (though we do so in the empirical work), meaning that the baseline utility for activity k on day l for person n is simply given by $\psi_{n,k,l} = e^{\delta_{k,d_{l_n}} + \varepsilon_{n,k,l}}$ where $\varepsilon_{n,k,l}$ is an extreme value error term for person n, activity k and l, and where d_{l_n} is the day type for day l for person n . In addition to making the δ parameters activity and day specific, we do the same for the γ parameters. This would thus lead to the estimation of KD different δ parameters and KD different γ parameters.

A model of the form above would allow for different utilities for the same product across days and also different shapes for the indifference curves, but would fail to capture correlation across activities and across days. We now instead define $\theta_n = \langle \delta_n, \gamma_n \rangle$ to be a vector combining the individual δ and γ parameters for person n, making these parameters individual-specific. Within θ_n, we would have that e.g. $\delta_n = \langle \delta_{n,1}, \ldots, \delta_{n,D} \rangle$, i.e. comprising itself different vectors where for example $\delta_{n,d} = \langle \delta_{n,d,1}, \ldots, \delta_{n,d,K} \rangle$, i.e. containing the constants used in the baseline utilities for activities on a day of type d by person n. The vector θ_n thus has $2KD$ elements, and we assume that it is distributed randomly across individuals, according to $\theta_n \sim f(\theta \mid \Omega)$.

Let $\theta_{n,d}$ be the subset of θ_n for days of type d. With $P_{n,l}(\theta_{n,d})$ giving the MDCEV probability (cf. Equation 3) for the consumption observed for individual n on day l (out of L_n), conditional on $\theta_{n,d}$. The unconditional probability for the observed sequence of day level consumptions for individual n is then given by:

$$P_n(\Omega) = \int_{\theta_n} \prod_{l=1}^{L_n} P_{n,l}\left(\theta_{n,d_{l_n}}\right) f(\theta \mid \Omega)\, \mathrm{d}\theta_n, \tag{4}$$

where d_{l_n} is the day type for observation l for respondent n, where the above notation ensures that the right subset of θ_n is used in $P_{n,l}\left(\theta_{n,d_{l_n}}\right)$.

By carrying out the integration over the distribution of θ_n at the person level rather than at the day level, we already capture correlation across days for the same person and for the same activity if those days are of the same type. However, the key flexibility arises if we allow for correlation between the different δ and γ parameters. Different possibilities arise. Let us assume without loss of generality that the multivariate distribution[1] used for θ_n is characterised by a mean for each element (i.e. every δ and γ term) along with a covariance matrix. In the closed form MDCEV model, we would be estimating the $2KD$ mean values, while all elements of the covariance matrix would be fixed to zero. In a model allowing for simple independent heterogeneity for each element, we would in addition estimate $2KD$ variances.

As a first step, we may want to focus on correlations in the baseline utilities for activities conducted on days of the same type. For day type d, we would now estimate K means for $\delta_{n,d}$, along with $\frac{K\cdot(K+1)}{2}$ covariance elements. This would for example allow us to understand in which way the baseline utilities for day type d are correlated with each other, e.g. whether a respondent who is more likely to take part in activity k_1 on day type d is also more likely to take part in activity k_2 on day type d. We may also want to allow for correlations across different day types in the baseline utilities for a given activity. This will imply the estimation of $K\frac{D\cdot(D-1)}{2}$ additional off-diagonal elements in the covariance matrix and would for example tell us whether respondents who are more likely to conduct leisure activities on a weekday are less likely to do so on

[1] Different distributions will likely apply for δ and γ, as discussed later on in our empirical work.

a Saturday. Allowing for correlations in baseline utilities within and across day types means the estimation of up to $\frac{KD\cdot(KD+1)}{2}$ elements of the covariance matrix. We can similarly allow for correlation between the individual elements of γ_n, which would give us some insights into how satiation is correlated across activities and day types.

The full level of flexibility of this mixed MDCEV model would involve the estimation of a full covariance matrix of $\frac{2KD\cdot(2KD+1)}{2}$ on top of the $2KD$ parameter means. This is substantial estimation issue which we address in our empirical work using Bayesian techniques.

Insights into substitution patterns can then be gained from the correlations. For example, negative correlation between the utilities of work activity on a weekday and that on Sunday implies a substitutive effect so that not working (or allocating less time to work) on weekdays may lead to working and/or allocating more time to work on Sunday (or vice versa). Similarly, a positive correlation might imply complementarity, so that working (and allocation more time to work) on a weekday may lead to working more on Sunday as well.

Conceptually, the above formulation belongs to the class of MDCEV models with multiple budget constraints. An individual has multiple 24 hour budget constraints, such that on day l, we have that $\sum_{k=1}^{K} x_i = 24$ and an overall $L \cdot 24$ budget constraint, such that $\sum_{l=1}^{L} \sum_{k=1}^{K} x_{kl} = L \cdot 24$ (if the individual level budgets are all met, then the multi-day one will be too). However, there is a subtle difference between our formulation and existing formulations with multiple budget constraints, such as those in Castro et al. (2012) and Pinjari and Sivaraman (2013). In most previous formulations, multiple budget constraints arise due to the use of multiple resources, such as time and money, for consuming a same choice alternative. Each activity would draw from both budgets.

In our formulation, however, activity participation on a day can draw only from the time available (24 hours) on that day. In other words, one cannot use time available on Sunday to work on a weekday. Since the 24 hour time budgets are not *fungible* across different days, as long as the utility functions are additively separable, it can be shown that the maximum multi-day utility derived by a person subject to multiple day-level constraints is the same as the sum (across all days) of maximum single-day utilities derived by the person subject to a single day's 24 hour time constraint. That is, $[Max(U), \text{subject to} \sum_{k=1}^{K} x_{kl} = 24 \,\forall l] = \sum_{l=1}^{L} [Max(u_l), \text{subject to} \sum_{k=1}^{K} x_{kl} = 24]$.

Therefore, conditional on the mixing distributions used in the specification, the multi-day time-use MDC choice probability may be derived as a simple product of single-day MDC choice probabilities, with as many single-day probabilities in the product as the number of days being modelled. In short, conditional on the mixing distributions, the multi-day time-use probability may be derived as a product of independent single-day MDCEV probabilities. The unconditional probability is simply an integral of this product over the mixing distribution.

2.4 Discussion

We now provide some brief contrasts between the two approaches discussed above. In comparison with the correlated, additive utility functions, the non-additive approach from Section 2.2 provides a more structural way to allow interactions. A major disadvantage, however, is that such non-additive utility models are very difficult to estimate and apply in practice; because the desirable optimisation properties of additive utility functions do not hold anymore. In addition, the additive utility specification easily allows correlations among baseline utility parameters as well as translation parameters, thereby differentiating substitution and/or complementarity effects in discrete choice from those in continuous choice dimensions. The non-additive utility approach, on the other hand, does not offer an easy way to disentangle interactions among the translation parameters from that of baseline utility parameters. This is because the interactions in the latter approach are between the utility terms, not between the parameters.

An advantage of the additive formulation in Section 2.3 is that it is simple, not very different from the standard mixed-MDCEV formulation, and therefore easy to implement for both estimation and application purposes. A drawback, however, as discussed in the next section, is that it is not easy to apply to predict substitution across different days. For example, if a person cannot work on a weekday due to some exogenous reasons, he/she will likely make up for the lost worktime by working more on a weekend day. Such substitution effects are not easy to accommodate when the model is applied for prediction despite the correlations retrieved during estimation, particularly when predictions are carried out separately for each day according to the estimated model. This is because the additively separable utility formulation does not incorporate explicit interactions between the utilities of time allocation across different days, except through correlations. Another disadvantage is that correlations between two utility functions may be either due to substitutive/complementarity relationships or simply due to common unobserved heterogeneity. The estimated correlation parameters typically capture a combined effect making it difficult to disentangle the source of correlation.

Given the difficulty of estimating and applying MDC models with non-additive utility functions, particularly those with multiple budget constraints, in this paper we employ the simpler model formulation discussed in Section 2.3 and explore various alternatives to apply such models for forecasting purposes, as discussed in Section 3. Of course, formulating, estimating, and applying non-additive, multi-day utility models while considering day-level constraints is an important avenue for future research.

3 Forecasting approach

We next turn our attention to forecasting from the above model, where our starting point is the efficient forecasting procedure of Pinjari and Bhat (2010), hereafter referred to as P&B, which we can rely on given the use of a generic α across all activities and a separate γ parameter for each inside good.

3.1 Base procedure adapted for random parameter case for a single day

We first illustrate the procedure for a specific individual n and a given day of type d, focussing on a time budget such that all *products* have unit prices (expressed in hours in most cases) and with a day level budget of 24 hours. The original approach uses sample enumeration (over R iterations) by relying on draws from the extreme value error terms of the model and we extend this to include the other random parameters in the model. The algorithm operates as follows.

Step 1 In iteration r, take a random vector of extreme value draws of length K ($\varepsilon_{n,d,r}$) and a realisation of the vector $\theta_{n,d}$ ($\theta_{n,d,r}$) taken from $f(\theta_{n,d} \mid \Omega_d)$ where Ω_d is the subset of Ω for day d.

Step 2 Assume that only the outside good is chosen by respondent n, setting the number of chosen goods M to 1.

Step 3 With the draws produced in Step 1, arrange the $K-1$ inside goods in descending order of $\psi_{n,d,k} = e^{\delta_{n,d,k}+\varepsilon_{n,d,k,r}}$, $\forall k$, and placing the outside good in first place, with $\psi_{n,d,1} = e^{\varepsilon_{n,d,1,r}}$.

Step 4 Using the ordering obtained in step 3, i.e. with $\psi_{n,d,m}$ being the m^{th} ranked activity[2], compute

$$\lambda = \left(\frac{24 + \sum_{m=2}^{M} \gamma_{n,d,m}}{\psi_{n,d,1}^{\frac{1}{1-\alpha}} + \sum_{m=2}^{M} \gamma_{n,d,m} \psi_{n,d,m}^{\frac{1}{1-\alpha}}} \right)^{\alpha-1}. \tag{5}$$

Step 5 If $\lambda > \psi_{n,d,M+1}$, again using the ordering from step 3, then conduct step 5a, else move to step 6.

Step 5a set the consumption of the outside good with draw r as

$$C_{n,d,1}^{(r)} = \frac{\psi_{n,d,1}^{\frac{1}{1-\alpha}} \left(24 + \sum_{m=2}^{M} \gamma_{n,d,m}\right)}{\psi_{n,d,1}^{\frac{1}{1-\alpha}} + \sum_{m=2}^{M} \gamma_{n,d,m} \psi_{n,d,m}^{\frac{1}{1-\alpha}}}. \tag{6}$$

Create a mapping index between the ordering of activities in the data (i.e. $k = 1, \ldots, K$) and the ordering of the activities from step 3. For example if the alternative ranked in position m in step 3 corresponds to the j^{th} activity in the data, then set $k_m = j$.

[2] Note that we only have correspondence between $k = 1$ and $m = 1$.

Next set the consumption of the remaining $M-1$ chosen activities as

$$C^{(r)}_{n,d,k_m} = \left(\frac{\psi_{n,d,m}^{\frac{1}{1-\alpha}} \left(24 + \sum_{l=2}^{M} \gamma_{n,d,l}\right)}{\psi_{n,d,1}^{\frac{1}{1-\alpha}} + \sum_{l=2}^{M} \gamma_{n,d,l} \psi_{n,d,l}^{\frac{1}{1-\alpha}}} - 1 \right) \gamma_{n,d,m},\ 1 < m \leq M. \tag{7}$$

If $M < K$, then set the consumption of alternatives above M, i.e. k_{M+1} and beyond in the mapping is set to zero.

Move to step 7.

Step 6 Set $M = M + 1$. If $M = K$, go to step 5a, else go to step 4.

Step 7 If $r = R$, stop the algorithm, else set $r = r + 1$ and go to step 1.

The application of the above algorithm produces a $R \cdot K$ dimensional matrix of consumption predictions for each individual n, and an average over the draws (i.e. rows of the $R \cdot K$ matrix) produces a vector of average predictions for this person.

3.2 Shortcomings of base algorithm when applied to multi-day data

Let us now consider a situation where we have estimated a model allowing for the types of heterogeneity and correlation discussed in Section 2.3 and wish to apply it to make a prediction not just for a single day but for a set of days for each person. In particular, and again without loss of generality, assume we want to make a prediction for a subset of L days for each person, where $L = D$, with one of each type $d = 1, \ldots, D$, for example a weekday, a Saturday and a Sunday.

The basic forecasting procedure in line with the estimated model would be very similar to that outlined in Section 3.1, with the only difference that it would be run L times for each individual respondent, i.e. once for each type of day in the above example. If multiple copies of the same day type were to be included, e.g. if making a prediction for two separate weekdays, then a further distinction would arise in that while the vector of extreme value draws in step 1 would differ across days, the same draws $\theta_{n,d,r}$ would be used for those days of the same type. This would lead to more similar predicted consumptions for those days of the same type, but with differences remaining due to the extreme value draws.

The forecasting process would account for the random heterogeneity across individuals as well as the correlations between the different parameters. For example, if the estimation reveals a positive correlation in the baseline utility for work on a weekday and on a Saturday, then this would be reflected in the forecasts, with those individuals who are predicted to work on a weekday also being more likely to have a prediction of working on a Saturday. However, a key aim of forecasting is to look at changes in behaviour. With the above approach, as the forecasts for each day are produced separately, then any changes in the scenario for say a weekday will not have any impact on the consumptions patterns observed for a Saturday. The forecasting approach is thus able to only account for the correlations between days in a base application but not for the effect this may have on redistribution across days in the case of a change from the base scenario.

3.3 Alternatives to base approach

It should be clear from the discussion in Section 3.2 that the application of the P&B routine at the day level makes it impossible for there to be any redistribution in activities across days, even if the model captures correlations that could reasonably be interpreted as grounds for such substitution to happen. Short of developing new model specifications (as discussed in Section 2.2) we now make initial attempts to adapt the approach from Section 3.1. Our discussions again focus on the case where we are making a prediction for a three day scenario covering a weekday (WD), Saturday (SAT) and Sunday (SUN), where the model estimation recovered separate parameters for these days, with correlations between them. We look at two possible approaches here, acknowledging that others are possible, and where for the second, we propose two versions.

3.3.1 Multi-day forecasting with separate outside goods

The forecasting routine in Section 3.1 works at the 24 hour level and needs to be applied separately for each day. With the three day example considered here, this approach would create, for person n, three separate $R{\cdot}K$ dimensional matrices of predicted consumptions. In our first departure from this approach, we now move away from the 24 budget and instead work with a budget of $L \cdot 24$ hours, where L is the number of days we make predictions for, in our case 3, such that the budget becomes 72. The algorithm will thus produce not three separate $R \cdot K$ matrices of predicted consumption, but a single $RxLK$ matrix, using $K' = LK$ activities, including L outside goods.

A number of changes are required to the base algorithm, as follows.

1. In step 1, we take a K' dimensional draw of extreme value errors, and similarly a $2K'$ dimensional vector of θ_n which now comprises δ and γ terms for different day types. If multiple days in the forecast are for the same day type, then the same draws from δ and γ would be used for those days. The subscript of day type is now dropped from γ and δ.

2. In step 2, we assume that all outside goods (i.e. one per day) are chosen and set $M = L$.

3. In step 3, the L inside goods are placed in first place, and the remaining $L \cdot (K-1)$ goods are put into descending order of utilities.

4. In step 4, Equation 5 is replaced by:

$$\lambda = \left(\frac{L \cdot 24 + \sum_{m=(L+1)}^{M} \gamma_{n,m}}{\sum_{m=1}^{L} \psi_{n,m}^{\frac{1}{1-\alpha}} + \sum_{m=(L+1)}^{M} \gamma_{n,m} \psi_{n,m}^{\frac{1}{1-\alpha}}} \right)^{\alpha-1} . \tag{8}$$

5. In step $5a$, we again start by creating a mapping index between the ordering of activities in our forecasting scenario (i.e. $k' = 1, \dots, K'$) and the ordering of the activities from step 3

   Equations 6 and Equation 7 are then replaced by:

$$C_{n,l}^{(r)} = \left(\frac{\psi_{n,l}^{\frac{1}{1-\alpha}} \left(L \cdot 24 + \sum_{m=(L+1)}^{M} \gamma_{n,m} \right)}{\sum_{m=1}^{L} \psi_{n,m}^{\frac{1}{1-\alpha}} + \sum_{m=(L+1)}^{M} \gamma_{n,m} \psi_{n,m}^{\frac{1}{1-\alpha}}} \right), \forall l < (L+1), \tag{9}$$

   and

$$C_{n,k'_m}^{(r)} = \left(\frac{\psi_{n,m}^{\frac{1}{1-\alpha}} \left(L \cdot 24 + \sum_{l=(L+1)}^{M} \gamma_{n,l} \right)}{\sum_{l=1}^{L} \psi_{n,l}^{\frac{1}{1-\alpha}} + \sum_{l=(L+1)}^{l} \gamma_{n,l} \psi_{n,l}^{\frac{1}{1-\alpha}}} - 1 \right) \gamma_{n,m}, \; L < m \leq M \tag{10}$$

At this point, we are left with a $R \cdot K'$ matrix of predicted consumptions. However, these predictions do not enforce the day level 24 budget constraint, only the total $L \cdot 24$ hour constraint. In other words, we have that $\sum_l C_{n,l}^{(r)} = L \cdot 24$, $\forall n, r$, but not necessarily (or likely) that e.g. $\sum_{l=1}^{K} C_{n,l}^{(r)} = 24$, $\forall n, r$. The raw predictions thus need to be rescaled as follows:

$$\begin{aligned}
\widehat{C_{n,l}^{(r)}} &= C_{n,l}^{(r)} \frac{24}{\sum_{k=1}^{K} C_{n,k}^{(r)}}, \; if\, 1 \leq l \leq K, \\
& \dots \\
\widehat{C_{n,l}^{(r)}} &= C_{n,l}^{(r)} \frac{24}{\sum_{k=(L-1)*K+1}^{K'} C_{n,k}^{(r)}}, \; if \; (L-1)\, K < l \leq K'
\end{aligned} \tag{11}$$

where this rescaling is performed prior to any averaging across draws.

3.3.2 Multi-day forecasting with single composite outside good

While the first proposed approach (Section 3.3.1) makes use of a $L \cdot 24$ hour budget with $K' = LK$ activities, including L outside goods, our second approach makes use of $K' = L\,(K-1) + 1$ activities, using a single composite outside good. Behaviourally, this is consistent with a situation where an individual determines over the course of several days how much time to invest across the different inside activities, with all the remainder going into the outside good.

To arrive at such a single composite outside good, consider the consumers problem of maximising $U = \sum_{l=1}^{L} \psi_{1l} \ln[x_l] + \sum_{l=1}^{L} \sum_{k=2}^{K} u_{kl}$,subject to L day-level time constraints $\sum_{k=1}^{K} x_{kl} = 24, \forall l = 1, 2, \dots, L$. In this utility function, consider $\sum_{l=1}^{L} \psi_{1l} \ln[x_{1l}]$, which is the sum of utility accrued from outside goods on all L days. Now, define $\psi_1 \ln[x_1]$ as the utility accrued from a single composite outside good x_1, with ψ_1 as its baseline utility term. For x_1 to serve as the composite outside good, the following condition should be satisfied:

$$\left[Max \left(\sum_{l=1}^{L} \psi_{1l} \ln[x_{1l}] \right), \text{subject to} \left(x_1 = \sum_{l=1}^{L} x_{1l} \right) \right] = \psi_1 \ln[x_1]. \tag{12}$$

That is, the maximum utility accrued from all the outside goods subject to a constraint that $x_1 = \sum_{l=1}^{L} x_{1l}$ should be equal to the utility accrued from the single composite outside good. One can go through KKT conditions of optimality to derive the optimal time allocations to the original outside goods as $x_{1l}^{*} = x_1 \times \frac{\psi_{1l}}{\sum_{t=1}^{L} \psi_{1t}}$. Plugging this expression into $\sum_{l=1}^{L} \psi_{1l} \ln[x_{1l}^{*}]$, we get the following expression for maximum utility from all essential goods:

$$Max \left(\sum_{l=1}^{L} \psi_{1l} \ln[x_{1l}] \right) = \left(\sum_{l=1}^{L} \psi_{1l} \right) \ln(x_1) + \sum_{l=1}^{L} \psi_{1l} \ln \left(\frac{\psi_{1l}}{\sum_{l=1}^{L} \psi_{1l}} \right), \tag{13}$$

where the log formulation arises from $\alpha = 0$.

Next, the consumers original utility maximisation problem mentioned at the beginning of this section may be expressed as:

$$Max \left[U = \left(\sum_{l=1}^{L} \psi_{1l} \right) \ln(x_1) + \sum_{l=1}^{L} \psi_{1l} \ln \left(\frac{\psi_{1l}}{\sum_{l=1}^{L} \psi_{1l}} \right) + \sum_{l=1}^{L} \sum_{k=2}^{K} u_{kl} \right], \tag{14}$$

subject to the following *single* time constraint: $x_1 + \sum_{l=1}^{L} \sum_{k=2}^{K} x_{kl} = L \cdot 24$.

This maximisation problem may further be rewritten as:

$$Max \left[U' = \left(\sum_{l=1}^{L} \psi_{1l} \right) \ln(x_1) + \sum_{l=1}^{L} \sum_{k=2}^{K} u_{kl} \right], \tag{15}$$

subject to $x_1 + \sum_{l=1}^{L} \sum_{k=2}^{K} x_{kl} = L \cdot 24$, because the term $\sum_{l=1}^{L} \psi_{1l} \ln \left(\frac{\psi_{1l}}{\sum_{l=1}^{L} \psi_{1l}} \right)$ is constant with respect to time allocations (which are the decision variables of the utility maximisation problem). Therefore, one can use the term $\left(\sum_{l=1}^{L} \psi_{1l} \right) \ln[x_1]$ to approximate the term $\left(\sum_{l=1}^{L} \psi_{1l} \ln[x_{1l}^{*}] \right)$ as a single composite good. To implement this for prediction purposes with a single composite outside good (representing multiple outside goods), we simulate a baseline utility that is equal to $\left(\sum_{l=1}^{L} \psi_{1l} \right)$ for the composite outside good.

In comparison with the approach with multiple outside goods, the changes required to the base algorithm are more limited, as follows.

- In step 1, we use the approach from the multi-day approach with separate outside goods when it comes to the $2K'$ dimensional vector of θ_n. For the extreme value terms, we also produce a vector of K' draws, however, the first three draws are then summed up to produce a composite error term for the composite outside good, leaving us with $L\,(K-1)+1$ error terms.

- This composite error term is then used in step 3 for the calculation of the baseline utility for the outside good.
- All other steps remain as in the P&B approach, with the difference that the budget is set to $L \cdot 24$ hours and that we work with $L(K-1)+1$ activities.

At this point, we are left with a $Rx(L(K-1)+1)$ matrix of predicted consumptions. Our first step consists of dividing the outside good up into L day-specific outside goods, again giving us a $R \cdot K'$ matrix of predicted consumptions. Two possibilities arise. In the first approach we evenly divide the prediction for the outside good across the L days. In the second approach, we recognise that some day types may see higher consumptions of the outside good and we make use of the sample level shares from the estimation data (i.e. what share of the outside good across L days is used on a given day) to guide the split across the L days[3]. Once this division has been performed, the same rescaling as described in Equation 11 is performed to ensure that the 24 hour level constraints are satisfied.

This approach thus allocates time to a single composite outside good and $L(K-1)$ inside goods, but where the utility of this outside good is greater than that of the single day outside goods in our other approach. A potential difference that arises between this approach and that using day-specific outside goods is that fewer corner solutions might arise for the inside goods with the composite outside good approach. This is the direct result of the way in which the P&B routine determines which activities to assign a non-zero consumption prior to determining the amount of continuous consumption, and the fact that the overall share (in terms of number of activities) for the inside goods is larger with the composite outside good approach ($L(K-1)$ out of $L(K-1)+1$ *vs* $L(K-1)$ out of $L \cdot K$).

3.4 Advantages and limitations

The discussions in this section have made clear the limitations of the standard P&B approach when used in the context of multi-day data where the model allows for correlations between days. With predictions produced separately for each day, no link is made between the consumptions for different days as a result of a change on a specific day. If for example the model retrieves positive correlation between the baseline utilities for working on a weekday and on a Saturday, then the forecasting approach will accommodate this correlation by predicting that someone who works more on a weekday will also work more on a Saturday (given that the random draws for δ are positively correlated). However, if an outside constraint were to affect working on a given weekday (leading to reduced consumption), this would have no impact on working on the Saturday, as that prediction comes from a separate 24 hour application of the model.

[3]We acknowledge that other ways to split the outside good are possible, for example relative to the utilities of the individual outside goods, but in practice, we found little differences across these approaches in terms of overall results.

A different situation arises with our proposed alternatives. The consumption across all days and activities is predicted in a single step, prior to rescaling. The correlations in the baseline scenario would be dealt with in the same way as in the single day approach (e.g. someone who is likely to work more on a weekday will also work more on a Saturday). However, with these approaches, if an outside constraint were to affect working on a given weekday, then for someone who has a higher utility of weekday work, the utility of Saturday work will also be larger and this activity will thus get a greater time allocation in the joint forecast across days.

While the above points suggest greater realism of our forecasting approaches, it should also be acknowledged that the alternative approaches put forward in this paper are somewhat *ad hoc* and present departures from the estimated model in a different direction, namely moving away from the 24 hour budget, and the enforcing this through rescaling. Our empirical work in the remainder of the paper provides initial insights into both the issues with performing single day forecasting and any detrimental effects that the move to the $L \cdot 24$ hour budget in forecasting might entail.

4 Empirical application

4.1 Data

This paper makes use of two well-known surveys in the transport literature, the German travel survey *Mobidrive* and the Chilean study *Communities in Concepción*. The use of two different datasets, with a different number of observations (days) for each participant, is instrumental for interpreting and validating our results.

The *Mobidrive* project conducted a six-week travel diary in the two German cities of Karlsruhe and Halle, with data collection taking place in the autumn of 1999. The availability of trip purpose allowed us to transform the travel diary into a time use diary. We only exploited two weeks of data, in particular we selected the second and third week recorded by respondents, to avoid bias due to any learning effects that may have occurred at the very beginning of the survey. Further information about the data collection protocol and the sample can be found in Axhausen et al. (2002).

The *Communities in Concepción* project conducted a rich data collection effort in 2012 in the Chilean city of Concepción. The survey included a time use diary that participants had to fill in for two days, one weekday and one weekend day. Further information about the survey and the sample can be found in Moore et al. (2013).

We use a subset of the overall *Mobidrive* sample: we only included respondents who do not fail to report any activity for more than 4 days over the two weeks used for the analysis, ending up with a sample of 223 respondents. For *Concepción*, we removed respondents with extensive lack of data/activity type, which resulted in a sample of 234 people. Corrections were applied in a few cases where the overall number of hours within a day would exceed 24h, for example if a respondent recorded the start of his/her activity/trip before midnight and the end during the early hours of the next day.

Both studies relied on paper-and-pencil diaries, where participants were free to specify the purpose of their trip (in the case of *Mobidrive*) or the activities that they had conducted (in the case of Concepción). For modelling purposes, these activities were subsequently grouped into a number of macro categories, depending on what participants reported and what was considered to be most relevant for the specific geographical and cultural context. Table 1 reports the sample averages for the discrete choice (percentage of people performing a given activity) and continuous choice (time invested in the different activities when this is performed, in hours). We present the statistics separately for weekday and weekend day, and in the case of Mobidrive we have sufficient information to also separate Saturday from Sunday. Some of the activities are present in both samples, while other categories are specific to one of the datasets.

The first activity, *Basic Needs*, includes sleeping, eating meals at home and spending time at home for everyday essential tasks. Everybody in the sample performs this activity every day, and this allows us to treat it as the "outside good" in our models.

Most of the categories in Table 1 are self-explanatory. *Work* refers to all work and work related activities. *School* refers to schooling and education activities. We choose to keep this separate from *Study*, defined for the Chilean context, as the latter did not include children, so this category refers to higher education or individual study. *Private business* includes personal errands, such as going to the bank, dentist, hairdresser (these correspond to the category *Services* in the Chilean context) with the addition of other personal activities. *Shopping* is an aggregated category in the Chilean data, although it is mainly believed to be groceries shopping, while it is split into daily and non-daily in *Mobidrive*.

4.2 Model specification and estimation

In the specification of our MDCEV models, we used a generic α across all activities including the outside good, which was found empirically to be very close to zero, along with a separate γ parameter for every inside good. Day-specific parameters were estimated to allow for day-level differences, while random heterogeneity was also accommodated both in the baseline utility constants and in the translation parameters. A base model and one including basic socio-demographic characteristics are estimated. In the latter case, we simply allowed for a deterministic shift in the baseline utility constants (δ) of each activity for male respondents (identical across days). This is by no means a full specification and was included purely for illustration purposes.

For the random parameters in *Mobidrive*, we use Normal distributions for the δ parameters and positive Lognormal distributions for the γ parameters (to ensure consistency with the modelling framework). For *Concepción*, we use negative Lognormals for δ and positive Lognormals for γ. The distributional assumptions for δ were arrived at by empirical testing of different possibilities. With both datasets, a full covariance matrix was estimated, to allow for correlation between all the model parameters.

As mentioned earlier, we make use of Bayesian estimation techniques to deal with the high number of parameters to be estimated, where we specifically make use of the

	Mobidrive						**Concepción**			
	Share participating			*Average time*			*Share participating*		*Average time*	
Activity	WD	SAT	SUN	WD	SAT	SUN	WD	WE	WD	WE
Basic Needs	100.00%	100.00%	100.00%	16.65	19.06	19.85	100.00%	100.00%	13.30	15.30
Work	41.75%	8.74%	3.59%	6.86	5.61	6.78	54.70%	13.68%	7.55	5.58
School	24.66%	2.02%	1.57%	5.30	3.01	5.85	-	-	-	-
Drop-off/Pick up	7.80%	7.85%	7.40%	0.71	0.42	0.54	14.10%	6.84%	0.43	0.72
Daily shopping	31.12%	36.10%	6.05%	0.60	0.69	0.29	-	-	-	-
Non daily shopping	14.53%	18.39%	2.47%	1.06	1.32	0.95	-	-	-	-
Social	24.08%	39.91%	42.15%	2.67	4.50	3.64	40.60%	55.56%	2.99	4.82
Leisure	19.46%	24.66%	29.15%	2.68	3.03	3.89	-	-	-	-
Private business	31.21%	16.82%	11.88%	1.15	1.42	0.98	-	-	-	-
Travel	97.26%	87.67%	79.82%	1.29	1.24	1.18	94.87%	81.62%	1.85	1.95
Family	-	-	-	-	-	-	11.97%	9.83%	3.57	3.41
Household obligations	-	-	-	-	-	-	26.50%	26.50%	4.48	5.39
Out-of-home recreation	-	-	-	-	-	-	12.82%	18.38%	2.46	2.74
In-home recreation	-	-	-	-	-	-	8.12%	11.97%	3.89	4.44
Services	-	-	-	-	-	-	16.24%	9.40%	1.66	2.51
Shopping	-	-	-	-	-	-	23.08%	26.50%	0.70	1.25
Study	-	-	-	-	-	-	20.51%	6.41%	4.24	4.02

Table 1: Average levels of continuous and discrete choice in the samples

RSGHB package (Dumont et al., 2015) from the R libraries (R Core Team, 2016). We use noninformative (diffuse) priors and make use of $300,000$ burn-in iterations to guarantee stable chains prior to averaging across iterations of the posteriors.

4.2.1 Mobidrive estimation results

The *Mobidrive* estimation results are presented in Table 2. As expected, we obtain a better log-likelihood (LL) for the model with the gender effect, where the improvement in fit is significant, as shown by a likelihood ratio test ($p \sim 1.6^{-5}$).

We first report the means and standard deviations across iterations for the Bayesian posteriors of both the mean and variances of the underlying Normal distribution (i.e. for the logarithm of γ). As mentioned earlier, the full covariance matrix between all random parameters was estimated, and this was used in the computation of correlations discussed below, where, because of space constraints, the full covariance matrix for each model is reported in an online appendix to this paper[4]. As discussed in Train (2001), these means and standard deviations of the posteriors have similar properties to maximum likelihood estimates and standard errors, respectively. Next to these, we provide the actual means (μ) and standard deviations (σ) for δ and γ, as used in the model. For *Mobidrive*, the δ parameters are normally distributed, and no transformation is needed, while the means and standard deviations for the lognormally distributed γ terms are calculated analytically from the means and standard deviations of the underlying Normals.

Looking at the resulting parameters for the base model, we observe that most of the baseline utility parameters are negative, mainly reflecting the discrete choice and indicating that the outside good (used as a base) is always "preferred", with respect to the inside goods, as everybody in the sample always chooses it. The value of the δ coefficients can also be affected by the continuous choice, so that it is possible to obtain positive δ coefficients for popular inside goods such as $Travel_{WD}$. This also motivates the use of a Normal distribution (instead of negative Lognormals, like in the *Concepción* case) for the δ parameters in this model. The γ parameters mainly describe the continuous choice, indicating that people spend most time in, i.e. they get less satiated by, *Work* on weekdays and on Sunday (this is reflected in the data on average time spent in different activities presented in Table 1), although the high standard deviations suggest large variation across people.

The results clearly show substantially different sensitivities during weekday and weekend days. In addition, they reveal substantial random heterogeneity (the σ parameters), highlighting that different people have different sensitivities, both in terms of the participation and in the time invested in different activities. Turning to deterministic heterogeneity, a comparison with the model including gender effects shows no substantial differences in the δ parameters on average. Larger changes are present in the γ parameters, for example in the case of *Drop-off/Pick-up* on Saturday, where we observe lower satiation in the model with the gender effect. The fixed shifts suggest that men are less

[4] http://www.stephanehess.me.uk/papers/Calastri_Hess_Pinjari_Daly_2017_online_appendix.pdf

likely to perform both daily and non daily shopping and more likely to perform leisure activities.

We next turn our attention to the correlations between the different model parameters, which we report in Table 3. For each of the 9 activities (inside goods), we present correlations between the δ and γ parameters across each pair of days j and k, both for the model with and without a gender effect. Interestingly, there are significant changes in the correlations between the two models, and it is not necessarily the case that the introduction of gender effects reduces the correlation in the unobserved heterogeneity, as might have been expected. Focussing on a few examples, we can see that the baseline utility constant for *Work* shows high levels of correlation across different days, with lower correlations between weekday and Sunday as well as Saturday and Sunday when the male effect is included in the model, as expected. Using the same activity as an example in the base model, we observe a negative correlation between the time invested in *Work* during weekdays and Saturdays, as well as weekdays and Sundays, while the correlation is positive between the two weekend days. Therefore, while people who work on weekdays are more likely to also work on weekends, the amount of time they spend working is likely to be negatively correlated.

Of course, we could not report all the possible correlations between all the model coefficients due to space constraints. In the lower part of the table we include some additional correlations that we considered of interest. Here, we specify the two activities and the respective days (in the order in which the activities are listed). As an example, we observe that there is a positive correlation between performing *Drop-off/Pick-up* and *Travel* on a Sunday, an effect which is stable in both the model with and without the gender effect. The correlation, although rather low, is positive also in terms of amount of time invested in these activities.

4.2.2 Concepción estimation results

The *Concepción* estimation results are presented in Table 4. As in the case of *Mobidrive*, we first present the statistics on the Bayesian posteriors before turning to the transformed parameters, where this time, as mentioned above, we use a negative Lognormal for the baseline utility constants and a positive Lognormal for the translation parameters.

The base model shows that the means of δ_{WD} and δ_{WE} for *Travel* are the least negative, reflecting the fact that this is the most popular activity after the outside good. For some activities, such as *Work*, the coefficients differ between weekday and weekend, showing that people are less likely to perform work activities during the weekend, while in the case of other activities, such as *Household Obligations* or *In-home recreation*, the difference is not as strong. A similar reasoning can be applied to the γ parameters to interpret the time investment in the different activities. The model with gender effects shows reduced utility for men for *Household Obligations*, with increased utility for *In-home recreation* and *Shopping*. As in the *Mobidrive* case, we can see that there is substantial variation not only across days but also across people in sensitivities. This is particularly pronounced in the case of *Household Obligations* and *Work*.

			Base model						Model with gender effect					
		Final LL	*-24,707.60*						*-24,673.78*					
		Parameters	*1,539*						*1,548*					
			Bayesian posteriors				**Resulting parameters**		**Bayesian posteriors**				**Resulting parameters**	
			μ_N		σ^2_N				μ_N		σ^2_N			
	Activity	**parameter**	mean	sd	mean	sd	μ	σ	mean	sd	mean	sd	μ	σ
parameters for baseline utilities	*Work*	δ_{WD}	-4.18	0.24	10.39	1.56	-4.18	3.22	-4.25	0.25	10.05	1.25	-4.25	3.17
		δ_{SAT}	-5.33	0.12	1.09	0.36	-5.33	1.04	-6.14	0.15	1.82	0.27	-6.14	1.35
		δ_{SUN}	-6.64	0.14	1.68	0.30	-6.64	1.29	-5.99	0.09	0.28	0.19	-5.99	0.53
		Δ_{male}	-	-	-	-	-	-	0.23	0.16	-	-	0.23	-
	School	δ_{WD}	-5.86	0.25	11.18	1.27	-5.86	3.34	-6.28	0.34	16.03	1.96	-6.28	4.00
		δ_{SAT}	-7.01	0.10	1.47	0.22	-7.01	1.21	-7.30	0.12	1.34	0.29	-7.30	1.16
		δ_{SUN}	-7.40	0.12	1.35	0.22	-7.40	1.16	-7.05	0.13	1.29	0.30	-7.05	1.13
		Δ_{male}	-	-	-	-	-	-	-0.33	0.28	-	-	-0.33	-
	Drop-off/ Pick-up	δ_{WD}	-5.99	0.14	2.08	0.33	-5.99	1.44	-6.12	0.16	2.40	0.40	-6.12	1.55
		δ_{SAT}	-5.93	0.13	1.11	0.36	-5.93	1.05	-5.66	0.11	0.40	0.14	-5.66	0.64
		δ_{SUN}	-5.92	0.15	1.14	0.19	-5.92	1.07	-5.63	0.11	0.95	0.20	-5.63	0.97
		Δ_{male}	-	-	-	-	-	-	-0.02	0.17	-	-	-0.02	-
	Daily shopping	δ_{WD}	-4.00	0.10	1.28	0.18	-4.00	1.13	-3.76	0.09	0.94	0.14	-3.76	0.97
		δ_{SAT}	-3.35	0.06	0.23	0.07	-3.35	0.48	-3.54	0.12	0.48	0.19	-3.54	0.69
		δ_{SUN}	-5.84	0.08	0.44	0.09	-5.84	0.66	-5.48	0.07	0.27	0.09	-5.48	0.52
		Δ_{male}	-	-	-	-	-	-	-0.32	0.11	-	-	-0.32	-
	Non daily shopping	δ_{WD}	-4.68	0.05	0.24	0.05	-4.68	0.49	-4.41	0.06	0.12	0.04	-4.41	0.35
		δ_{SAT}	-4.48	0.05	0.17	0.05	-4.48	0.42	-4.29	0.08	0.34	0.07	-4.29	0.58
		δ_{SUN}	-6.64	0.08	0.14	0.04	-6.64	0.38	-6.60	0.08	0.34	0.10	-6.60	0.58
		Δ_{male}	-	-	-	-	-	-	-0.44	0.10	-	-	-0.44	-
	Social	δ_{WD}	-4.21	0.08	0.78	0.13	-4.21	0.88	-4.24	0.11	0.83	0.13	-4.24	0.91
		δ_{SAT}	-3.47	0.09	0.46	0.11	-3.47	0.68	-3.64	0.08	0.46	0.10	-3.64	0.68
		δ_{SUN}	-3.45	0.08	0.51	0.11	-3.45	0.71	-3.38	0.10	0.22	0.06	-3.38	0.47
		Δ_{male}	-	-	-	-	-	-	0.11	0.11	-	-	0.11	-
	Leisure	δ_{WD}	-4.76	0.12	1.74	0.28	-4.76	1.32	-4.82	0.14	1.54	0.23	-4.82	1.24
		δ_{SAT}	-4.20	0.09	0.81	0.16	-4.20	0.90	-4.51	0.10	0.64	0.18	-4.51	0.80
		δ_{SUN}	-4.09	0.11	0.96	0.18	-4.09	0.98	-4.10	0.13	0.84	0.15	-4.10	0.92
		Δ_{male}	-	-	-	-	-	-	0.28	0.15	-	-	0.28	-
	Private business	δ_{WD}	-3.79	0.07	0.55	0.09	-3.79	0.74	-3.80	0.08	0.57	0.11	-3.80	0.76
		δ_{SAT}	-4.60	0.10	0.31	0.11	-4.60	0.56	-4.51	0.09	0.18	0.07	-4.51	0.43
		δ_{SUN}	-5.05	0.11	0.55	0.10	-5.05	0.74	-5.05	0.16	0.24	0.08	-5.05	0.49
		Δ_{male}	-	-	-	-	-	-	-0.01	0.10	-	-	-0.01	-
	Travel	δ_{WD}	0.87	0.09	0.25	0.06	0.87	0.50	0.78	0.08	0.22	0.05	0.78	0.47
		δ_{SAT}	-0.80	0.08	0.11	0.04	-0.80	0.33	-0.67	0.09	0.49	0.16	-0.67	0.70
		δ_{SUN}	-1.35	0.11	0.45	0.11	-1.35	0.67	-1.36	0.09	0.30	0.08	-1.36	0.55
		Δ_{male}	-	-	-	-	-	-	0.08	0.06	-	-	0.08	-
translation parameters	*Work*	γ_{WD}	1.63	0.14	3.03	0.51	23.28	103.15	1.73	0.14	3.09	0.49	26.42	121.13
		γ_{SAT}	1.23	0.09	0.33	0.09	4.03	2.53	0.87	0.09	1.11	0.24	4.18	5.98
		γ_{SUN}	1.93	0.14	0.62	0.17	9.41	8.71	1.92	0.09	0.46	0.12	8.62	6.57
	School	γ_{WD}	0.75	0.12	0.72	0.18	3.04	3.11	1.15	0.14	1.65	0.34	7.18	14.73
		γ_{SAT}	0.16	0.12	0.52	0.12	1.53	1.26	-0.01	0.10	0.68	0.14	1.39	1.37
		γ_{SUN}	1.67	0.09	0.35	0.08	6.34	4.08	1.88	0.16	0.85	0.20	9.97	11.50
	Drop-off/ Pick-up	γ_{WD}	-0.99	0.08	0.35	0.06	0.44	0.28	-1.09	0.11	0.58	0.13	0.45	0.40
		γ_{SAT}	-1.86	0.11	1.13	0.21	0.27	0.40	-0.55	0.11	0.44	0.12	0.72	0.53
		γ_{SUN}	-0.22	0.16	1.82	0.33	1.99	4.53	-0.94	0.13	1.04	0.23	0.65	0.89
	Daily shopping	γ_{WD}	-1.03	0.06	0.32	0.06	0.42	0.26	-1.17	0.09	0.20	0.09	0.34	0.16
		γ_{SAT}	-1.17	0.06	0.10	0.04	0.33	0.10	-1.07	0.06	0.13	0.06	0.36	0.13
		γ_{SUN}	-1.16	0.13	0.34	0.12	0.37	0.24	-1.96	0.16	0.38	0.11	0.17	0.12
	Non daily shopping	γ_{WD}	-0.46	0.09	0.07	0.03	0.66	0.18	-0.50	0.08	0.07	0.02	0.63	0.17
		γ_{SAT}	-0.34	0.08	0.56	0.10	0.94	0.81	-0.36	0.07	0.25	0.05	0.79	0.42
		γ_{SUN}	0.53	0.10	0.43	0.09	2.12	1.56	-0.09	0.10	0.62	0.14	1.24	1.14
	Social	γ_{WD}	0.55	0.06	0.18	0.08	1.90	0.86	0.72	0.13	0.23	0.05	2.31	1.17
		γ_{SAT}	1.19	0.07	0.11	0.05	3.48	1.21	1.09	0.10	0.21	0.06	3.31	1.61
		γ_{SUN}	0.64	0.12	0.08	0.03	1.98	0.59	0.52	0.07	0.07	0.03	1.73	0.46
	Leisure	γ_{WD}	0.37	0.14	0.12	0.05	1.53	0.54	0.56	0.07	0.13	0.04	1.87	0.69
		γ_{SAT}	0.88	0.08	0.11	0.06	2.54	0.87	0.72	0.07	0.23	0.10	2.30	1.16
		γ_{SUN}	1.04	0.10	0.20	0.07	3.13	1.49	0.84	0.09	0.12	0.04	2.47	0.88
	Private business	γ_{WD}	-0.74	0.06	0.14	0.07	0.51	0.20	-0.81	0.07	0.16	0.03	0.48	0.20
		γ_{SAT}	-0.16	0.12	0.61	0.10	1.16	1.06	-0.71	0.06	0.26	0.07	0.56	0.31
		γ_{SUN}	-0.93	0.15	1.15	0.19	0.70	1.03	-1.34	0.10	0.46	0.12	0.33	0.25
	Travel	γ_{WD}	-3.63	0.09	0.09	0.03	0.03	0.01	-3.56	0.09	0.09	0.04	0.03	0.01
		γ_{SAT}	-2.29	0.09	0.07	0.04	0.11	0.03	-2.52	0.07	0.29	0.08	0.09	0.05
		γ_{SUN}	-2.00	0.07	0.24	0.08	0.15	0.08	-1.95	0.05	0.13	0.04	0.15	0.06

Table 2: Mobidrive - Estimation results

		Base model		With gender	
Activity	**Day**	$corr(\delta_j, \delta_k)$	$corr(\gamma_j, \gamma_k)$	$corr(\delta_j, \delta_k)$	$corr(\gamma_j, \gamma_k)$
	WD-SAT	0.71	-0.13	0.93	-0.13
Work	WD-SUN	0.61	-0.10	0.17	0.16
	SAT-SUN	0.77	0.54	0.19	-0.19
	WD-SAT	0.67	0.64	0.62	-0.05
School	WD-SUN	0.85	0.46	0.67	0.37
	SAT-SUN	0.88	-0.03	0.64	-0.37
	WD-SAT	0.81	-0.14	0.73	0.13
Drop-off/ Pick-up	WD-SUN	0.68	0.40	0.74	0.45
	SAT-SUN	0.38	-0.12	0.57	0.67
	WD-SAT	0.67	0.47	0.80	0.74
Daily shopping	WD-SUN	0.60	0.61	0.69	0.48
	SAT-SUN	0.48	0.16	0.49	0.23
	WD-SAT	0.09	0.05	0.48	-0.37
Non daily shopping	WD-SUN	0.17	-0.07	0.39	0.34
	SAT-SUN	0.32	-0.08	0.03	-0.19
	WD-SAT	0.85	0.36	0.77	0.37
Social	WD-SUN	0.84	0.20	0.77	-0.37
	SAT-SUN	0.79	0.18	0.66	-0.11
	WD-SAT	0.90	0.29	0.91	0.05
Leisure	WD-SUN	0.87	0.36	0.88	0.47
	SAT-SUN	0.85	0.42	0.92	0.26
	WD-SAT	0.76	0.58	0.76	0.61
Private business	WD-SUN	0.66	0.56	0.46	0.31
	SAT-SUN	0.34	0.67	0.28	0.50
	WD-SAT	0.58	0.26	-0.01	-0.43
Travel	WD-SUN	0.52	0.47	-0.08	-0.19
	SAT-SUN	0.62	0.23	0.75	0.35
Daily shopping & Non daily shopping	SUN-SUN	0.39	-0.25	0.69	0.64
Drop-off/ Pick-up & Travel	SUN-SUN	0.79	0.13	0.75	0.15
Leisure & Travel	SUN-SUN	0.58	0.33	0.60	0.08
School & Daily shopping	WD-WD	-0.64	-0.39	-0.70	-0.35
	WD-SAT	-0.42	-0.37	-0.66	-0.26
School & Non daily shopping	WD-WD	-0.60	0.11	-0.70	0.09
Social & Private business	SAT-SUN	0.25	-0.13	0.61	-0.41
Social & Travel	SAT-SUN	0.61	0.24	0.69	0.39
Work & School	WD-WD	-0.55	-0.04	-0.52	-0.04
Work & Social	SUN-SUN	-0.28	0.32	-0.53	0.45

Table 3: Mobidrive - Key correlations

Selected correlations between the model parameters are reported in Table 5. We observe high positive correlations in the δ parameters for *Household Obligations*, *Study* and *In-home recreation*, with the first two being stable across the two models. The correlation for *Work* is positive but not as high as in the *Mobidrive* data, while the negative correlation in the γ between the weekday and the weekend day is relatively weak.

As in the *Mobidrive* case, we present some additional correlations between the coefficients related to different activities, both for the same day and across different days.

For example, we find a positive correlation in the baseline utility constant of *Drop-off/ Pick-up* and *Work* on a weekday, possibly suggesting that the same people perform both activities on that day, although the time spent is negatively correlated. While the signs are maintained, the magnitude of the effects is again slightly stronger in the model with the gender effect. On the contrary, a negative correlation in the δ parameter on the same day (weekday) is observed for *Out-of-home recreation* and *Work*; this could be due to the fact that these are both activities that are quite time-intensive and so, if performed, require a relatively high time investment.

4.3 Forecasting examples

We next turn to the results of model application using the different forecasting approaches presented in Section 3. In what follows, for ease of presentation, we use the following labelling for the different approaches:

Approach A The base approach using predictions at the day level, with a separate 24 hour budget for each day (see Section 3.1)

Approach B Our approach using multi-day forecasting with separate outside goods for each day, followed by rescaling to satisfy the 24 constraint for each day (see Section 3.3.1)

Approach C1 Our approach using multi-day forecasting with a single composite outside good, followed by rescaling to satisfy the 24 constraint for each day, where the outside good is split evenly into L parts before rescaling (see Section 3.3.2)

Approach C2 Like approach C1, but where the split of the composite outside good before rescaling takes into account the split from the data used in estimation (see Section 3.3.2)

As discussed in detail below, we look at forecasts for 3 or 4 days with *Mobidrive*, and 2 days with *Concepción*. We apply the different forecasting approaches using 250 MLHS draws (Hess et al., 2006) per individual.

4.3.1 Preliminary appraisal

As discussed in Section 3, our proposed alternatives to the base approach are not theoretically in line with the estimated model. From that perspective, it becomes important to test the reasonableness of the forecasts not just in terms of the implied substitution patterns (which will almost surely be better for our proposed approaches given that they allow for cross-day substitution) but also in terms of the *quality* of the base forecasts, i.e. predictions under a *do nothing* scenario. We do this by looking both at how likely the predictions are according to the estimated model (i.e. calculating the log-likelihood for the prediction) and what the total utility of the predicted consumptions is across all individuals. The use of the log-likelihood could be affected by a greater share of corner

			Base model						Model with gender effect					
		Final LL	-5,051.03						-5,030.01					
		Parameters	1,034						1,045					
			Bayesian posteriors				Resulting parameters		Bayesian posteriors				Resulting parameters	
			μ_N		σ^2_N				μ_N		σ^2_N			
	Activity	**parameter**	mean	sd	mean	sd	μ	σ	mean	sd	mean	sd	μ	σ
parameters for baseline utilities	*Work*	δ_{WD}	0.97	0.05	0.06	0.03	-2.72	0.66	1.00	0.05	0.08	0.04	-2.84	0.29
		δ_{WE}	1.55	0.04	0.03	0.02	-4.80	0.87	1.58	0.05	0.04	0.02	-4.93	0.19
		Δ_{male}	-	-	-	-	-	-	0.29	0.20	-	-	-	-
	Drop-off/ Pick-up	δ_{WD}	1.51	0.04	0.03	0.02	-4.60	0.85	1.51	0.05	0.04	0.03	-4.63	0.21
		δ_{SAT}	1.72	0.05	0.03	0.02	-5.70	1.07	1.73	0.05	0.04	0.02	-5.74	0.20
		Δ_{male}	-	-	-	-	-	-	0.07	0.32	-	-	-	-
	Social	δ_{WD}	1.12	0.04	0.03	0.01	-3.12	0.55	1.16	0.04	0.02	0.01	-3.22	0.15
		δ_{WE}	0.97	0.04	0.03	0.01	-2.68	0.46	1.03	0.04	0.03	0.02	-2.85	0.19
		Δ_{male}	-	-	-	-	-	-	0.31	0.16	-	-	-	-
	Travel	δ_{WD}	-1.29	0.15	0.03	0.02	-0.28	0.05	-1.82	0.15	0.03	0.03	-0.16	0.18
		δ_{WE}	0.52	0.09	0.04	0.03	-1.72	0.37	0.40	0.08	0.13	0.07	-1.59	0.36
		Δ_{male}	-	-	-	-	-	-	0.20	0.14	-	-	-	-
	Family	δ_{WD}	1.53	0.05	0.03	0.02	-4.70	0.83	1.56	0.06	0.04	0.03	-4.84	0.20
		δ_{WE}	1.64	0.06	0.03	0.02	-5.20	0.84	1.61	0.06	0.03	0.01	-5.08	0.16
		Δ_{male}	-	-	-	-	-	-	-0.14	0.33	-	-	-	-
	Household obligations	δ_{WD}	1.40	0.07	0.23	0.10	-4.55	2.30	1.27	0.07	0.14	0.05	-3.82	0.38
		δ_{WE}	1.42	0.05	0.13	0.05	-4.39	1.63	1.33	0.05	0.09	0.05	-3.95	0.30
		Δ_{male}	-	-	-	-	-	-	-0.81	0.28	-	-	-	-
	Out-of-home recreation	δ_{WD}	1.53	0.05	0.05	0.03	-4.77	1.13	1.60	0.05	0.06	0.03	-5.11	0.25
		δ_{WE}	1.49	0.06	0.05	0.03	-4.54	1.02	1.49	0.04	0.04	0.02	-4.54	0.20
		Δ_{male}	-	-	-	-	-	-	0.31	0.26	-	-	-	-
	In-home recreation	δ_{WD}	1.64	0.06	0.04	0.03	-5.27	1.11	1.81	0.10	0.08	0.04	-6.35	0.28
		δ_{WE}	1.61	0.08	0.05	0.04	-5.16	1.18	1.73	0.08	0.08	0.04	-5.84	0.28
		Δ_{male}	-	-	-	-	-	-	0.91	0.37	-	-	-	-
	Services	δ_{WD}	1.43	0.04	0.02	0.01	-4.24	0.55	1.46	0.05	0.02	0.01	-4.33	0.13
		δ_{WE}	1.62	0.04	0.02	0.01	-5.11	0.78	1.65	0.06	0.04	0.03	-5.33	0.20
		Δ_{male}	-	-	-	-	-	-	0.20	0.28	-	-	-	-
	Shopping	δ_{WD}	1.34	0.04	0.02	0.01	-3.87	0.60	1.41	0.04	0.03	0.02	-4.16	0.17
		δ_{WE}	1.34	0.05	0.03	0.02	-3.87	0.64	1.38	0.04	0.04	0.02	-4.03	0.19
		Δ_{male}	-	-	-	-	-	-	0.40	0.21	-	-	-	-
	Study	δ_{WD}	1.41	0.06	0.05	0.02	-4.21	0.93	1.44	0.05	0.05	0.03	-4.32	0.22
		δ_{WE}	1.79	0.09	0.06	0.04	-6.16	1.58	1.74	0.08	0.05	0.02	-5.85	0.22
		Δ_{male}	-	-	-	-	-	-	0.05	0.30	-	-	-	-
translation parameters	*Work*	γ_{WD}	1.74	0.16	0.07	0.08	5.91	1.64	1.64	0.13	0.07	0.05	5.34	1.47
		γ_{WE}	1.56	0.15	0.03	0.02	4.83	0.89	2.05	0.23	0.06	0.03	7.98	1.91
	Drop-off/ Pick-up	γ_{WD}	-1.76	0.10	0.04	0.02	0.18	0.03	-1.54	0.13	0.04	0.02	0.22	0.04
		γ_{SAT}	-0.76	0.18	0.03	0.02	0.47	0.09	-1.07	0.14	0.06	0.04	0.35	0.09
	Social	γ_{WD}	0.62	0.11	0.03	0.03	1.89	0.34	0.40	0.14	0.03	0.02	1.52	0.26
		γ_{WE}	0.49	0.09	0.04	0.03	1.65	0.32	0.75	0.06	0.03	0.02	2.14	0.38
	Travel	γ_{WD}	-2.08	0.11	0.02	0.01	0.13	0.02	-2.26	0.10	0.03	0.02	0.11	0.02
		γ_{WE}	-0.92	0.18	0.04	0.03	0.41	0.09	-1.24	0.08	0.04	0.03	0.29	0.06
	Family	γ_{WD}	0.55	0.15	0.03	0.02	1.76	0.30	0.92	0.10	0.04	0.03	2.56	0.51
		γ_{WE}	1.45	0.15	0.07	0.04	4.42	1.16	0.74	0.16	0.03	0.02	2.12	0.37
	Household obligations	γ_{WD}	0.90	0.10	0.03	0.02	2.49	0.41	1.08	0.07	0.04	0.03	3.01	0.61
		γ_{WE}	1.38	0.25	0.05	0.04	4.08	0.92	1.79	0.16	0.05	0.03	6.11	1.32
	Out-of-home recreation	γ_{WD}	0.48	0.11	0.04	0.03	1.65	0.32	-0.34	0.09	0.05	0.03	0.73	0.16
		γ_{WE}	0.69	0.08	0.03	0.02	2.02	0.36	0.68	0.12	0.05	0.04	2.02	0.46
	In-home recreation	γ_{WD}	0.91	0.10	0.04	0.03	2.52	0.52	1.44	0.16	0.05	0.03	4.31	0.94
		γ_{WE}	1.58	0.10	0.03	0.02	4.92	0.92	1.18	0.14	0.04	0.03	3.30	0.64
	Services	γ_{WD}	0.13	0.12	0.04	0.03	1.16	0.23	-0.02	0.16	0.04	0.03	1.00	0.20
		γ_{WE}	1.08	0.16	0.07	0.05	3.05	0.80	0.77	0.08	0.03	0.02	2.19	0.37
	Shopping	γ_{WD}	-0.88	0.16	0.03	0.02	0.42	0.08	-0.77	0.14	0.04	0.03	0.47	0.10
		γ_{WE}	-0.37	0.19	0.04	0.02	0.70	0.13	-0.16	0.14	0.04	0.03	0.87	0.18
	Study	γ_{WD}	1.08	0.12	0.03	0.02	2.98	0.53	0.93	0.14	0.06	0.03	2.61	0.64
		γ_{WE}	1.34	0.09	0.04	0.04	3.90	0.83	0.39	0.13	0.09	0.08	1.55	0.49

Table 4: Concepción - Estimation results

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		Base model		With gender	
Activity	**Day**	$corr(\delta_j, \delta_k)$	$corr(\gamma_j, \gamma_k)$	$corr(\delta_j, \delta_k)$	$corr(\gamma_j, \gamma_k)$
Work	WD-WE	0.46	-0.16	0.48	-0.14
Drop-off/ Pick-up	WD-WE	0.12	0.10	0.43	-0.36
Social	WD-WE	0.11	0.25	0.11	-0.01
Travel	WD-WE	-0.10	-0.17	0.28	-0.16
Family	WD-WE	0.02	0.30	0.02	0.02
Household obligations	WD-WE	0.85	-0.10	0.84	-0.21
Out-of-home recreation	WD-WE	0.60	0.06	0.56	-0.28
In-home recreation	WD-WE	0.57	-0.04	0.73	-0.12
Services	WD-WE	0.01	0.22	-0.22	-0.29
Shopping	WD-WE	0.01	-0.11	0.30	0.31
Study	WD-WE	0.67	-0.07	0.65	0.24
Drop-off/ Pick-up & Work	WD-WD	0.44	-0.43	0.59	-0.26
Work & Household obligations	WD-WE	0.55	-0.31	0.58	0.34
Shopping & Travel	WE-WE	0.13	0.27	0.52	0.30
Drop-off/ Pick-up & Family	WD-WD	0.44	-0.30	0.51	0.20
Work & Shopping	WD-WE	0.39	-0.31	0.36	0.13
Household obligations & Out-of-home recreation	WE-WE	-0.62	-0.06	-0.62	-0.25
Out-of-home recreation & Work	WD-WD	-0.59	-0.13	-0.64	0.34
Household obligations & Out-of-home recreation	WD-WD	-0.49	-0.14	-0.65	-0.09
Study & Work	WD-WD	-0.56	0.03	-0.66	0.34
In-home recreation & Travel	WE-WE	-0.50	-0.25	-0.72	0.41

Table 5: Concepción - Key correlations

solutions in some forecasts, which is likely to inflate it (as they are easy to explain) and this is the reason for looking at other measures too. For the utility calculations, a key reason is to see which approach provides the maximum utility. One would expect Approach A to yield forecasts with maximum utility according to the estimated model. The second reason is to assess how suboptimal the utility of time allocations from the alternative approaches is (since they are not based on the estimated model). We are able to perform this calculation on the raw forecasts which do not enforce the day-level constraints as well as on the rescaled ones. In addition, we compare the predicted discrete and continuous consumptions to those from the estimation data by means of a root mean square error (RMSE).

An overview of the findings is given in Table 6. In terms of LL, we observe that approaches C1 and C2 give very similar fit, where C1 is always better than C2. More importantly, both C1 and C2 outperform approaches A and B, where B is always better than A, though it is closer to A than C1 and C2 in the case of *Mobidrive*. While these differences in LL provide some reassurance about the use of our approaches, they raise the question of why they perform better than the approach which is consistent with the estimated model (A). The answer would seem to lie in the fact that while approach A explicitly recognises the 24 hour constraint, it fails to accommodate the correlation across different days. This *disadvantage* seems to outweigh the *advantage* approach A has over B and C in terms of theoretical consistency. The advantage of approach C1 and C2 seems to be down the use of a composite outside good which leads to fewer corner solutions for the inside goods than the use of separate outside goods for each day in approach B.

The latter predicts more corner solutions than those actually present in the data, while approaches C1 and C2 produce a prediction which is closer to what is observed in reality. In the *Mobidrive* data, we can compute the average of the discrete choice (which for each person and each activity on a given day takes value 0 or 1) across all people and all inside goods at the 3 day level, which gives 0.27. This is equivalent to stating that on average across all people and all inside activities, the probability of conducting an inside activity is 27% (or conversely a probability of 73% of a corner solution). Looking at this share as predicted by the different forecasting approaches, we see that approach A predicts this to be 0.24, while it is 0.19 with approaches C1 and C2. However, with approach B, it is only 0.12, with similar patterns in the other application runs. This implies that approach B predicts on average a share of corner solutions of 0.88, which is higher than in the case of the other approaches. This is in line with the earlier hypothesis in Section 3.3.2.

These differences in the predicted shares of respondents participating in different activities leads us directly to the RMSE measures, where we look at the differences between the predicted values and the ones actually observed in the data, for both discrete and continuous. For *Mobidrive*, we see that the C approaches perform best for the continuous consumption, and A performs worst. B is not too different from C1 and C2 in the continuous, but worse for discrete. For *Concepción*, approaches C1, C2 and B however perform badly for the continuous choice as they in fact overestimate the consumption of the outside good across the two days.

We finally turn our attention to the total utilities for the forecast. We see that these are similar for the different approaches, but are slightly better for approach A in all cases, which reflects the fact that this approach is in line with the estimated model. However, the loss in utility for approaches B and C is small. It is interesting to see that the utility of the different approaches before the rescaling to meet the 24 hour constraint is applied is much higher. Approach B is greatly penalised by the rescaling. An important area for future work is to look at ways for the forecasting to benefit from the $L \cdot 24$ allocation while respecting day level constraints and thus not requiring rescaling.

4.3.2 Mobidrive - Forecasting results

In this section, we present detailed results for the different forecasting approaches applied to the *Mobidrive* dataset. Table 7 presents the detailed results for the application with 3 days, including a Friday, a Saturday and a Sunday, while in Table 8 we add an additional weekday. In addition to the *do nothing* scenario, we look at a forecast scenario in which a change is made. In the absence of detailed explanatory variables, we look at a situation where working on a Friday becomes less attractive for some reason, and where we subtract half the absolute mean of the randomly distributed δ for *Work*. This implies that people will not gain as much utility from *Work* on that day. In practical terms, this could be associated to a situation when someone has some errands to perform on a Friday, so he/she could decide to work less or not work at all (i.e. have a corner solution). The Tables show percentage changes (averages across the sample) in the discrete consumption,

				LL for forecast using estimated model	*Utility of consumption*	*Utility of consumption before rescaling*	*RMSE vs data for continuous consumption*	*RMSE vs data for discrete consumption*
Mobidrive	3 days	Base model	Approach A	-2,043.40	155,510.88	-	0.57	0.03
			Approach B	-1,921.85	154,999.52	205,900.88	0.32	0.16
			Approach C1	-1,559.39	155,297.61	158,614.68	0.29	0.09
			Approach C2	-1,564.38	155,296.55	159,850.18	0.28	0.09
		With gender	Approach A	-1,987.85	154,963.08	n/a	0.52	0.04
			Approach B	-1,905.62	154,362.57	204,904.84	0.36	0.17
			Approach C1	-1,521.64	154,759.63	157,715.96	0.32	0.09
			Approach C2	-1,525.80	154,758.41	158,951.47	0.32	0.09
	4 days	Base model	Approach A	-2,792.49	167,178.24	n/a	0.56	0.03
			Approach B	-2,614.20	165,969.48	233,066.88	0.43	0.19
			Approach C1	-1,893.44	166,798.58	170,831.69	0.41	0.10
			Approach C2	-1,900.85	166,796.52	170,473.57	0.42	0.10
		With gender	Approach A	-2,736.61	168,744.14	n/a	0.52	0.04
			Approach B	-2,612.34	167,235.39	234,685.64	0.47	0.19
			Approach C1	-1,871.73	168,366.19	172,794.46	0.45	0.10
			Approach C2	-1,877.33	168,364.04	172,436.34	0.45	0.10
Concepcion	2 days	Base model	Approach A	-2,080.13	39,172.62	n/a	0.24	0.05
			Approach B	-1,724.88	38,760.58	45,373.11	0.50	0.14
			Approach C1	-1,710.35	38,999.36	42,524.61	0.55	0.10
			Approach C2	-1,714.65	38,998.62	42,475.57	0.55	0.10
		With gender	Approach A	-2,050.42	43,266.13	n/a	0.25	0.05
			Approach B	-1,750.78	42,957.59	50,298.50	0.47	0.14
			Approach C1	-1,694.62	43,091.81	47,341.17	0.52	0.09
			Approach C2	-1,698.92	43,091.15	47,292.13	0.52	0.09

Table 6: Overview of the forecasting approaches

i.e. in the number of people who perform the activity, and changes in the continuous consumption, i.e. in time invested in the different activities (average time across all people, not only those who perform the activity).

Approach A is the base forecasting procedure, which forecasts according to the model, i.e. at the 24 hour level. There is a 51.17% reduction in the share of people who go to work on Friday (base model), while we see an increase in the share of people performing other activities. It is worth remembering that there is no change in the outside good as it is always consumed by everyone in the sample, by definition. When it comes to the continuous choice, we observe redistribution within the day: in the base model, there is, on average, a 73.31% decrease in the time allocated to *Work* and an increase in time allocated to other activities; the vast majority of this decrease comes from those people who move to corner solutions, and there are far smaller changes in the time use for those who do not change their discrete choice. This increase is not equal across activities and is driven in part by the estimated correlations. Given that the forecasting is performed at the day level, we do not observe any substitution across days.

Moving on to the other approaches, we see similar percentage reductions in the share of people performing the activity as well as in the time allocation to *Work* on the Friday, but also a redistribution to *Work* on Saturday and Sunday, implying that if unable to work on Friday, people will compensate on the weekend days. Approaches B, C1 and C2 all show a similar reallocation to Saturday than Sunday in the base model, while in the model with gender effects, the reallocation is much higher to Saturday than to Sunday. These results are in line with the correlations across the different days for *Work* presented in Table 3.

As explained in Section 3.2, we can include more than one day of the same type, in our case weekdays. We perform the forecasting exercise with four days, assuming we include a Thursday, a Friday, a Satuday and a Sunday. In this case, we apply the change to the baseline utility of *Work* on the Thursday. The average percentage reduction in time spent at work on the Thursday, similarly to the case of the Friday in the 3-day example, is higher than 70% (cf. Table 8). We observe here that the substitution to Saturday and Sunday is much higher than the one to Friday. This is due to the fact that Thursday and Friday are two identical days prior to the change in the baseline utility (and notwithstanding the use of different extreme value draws), and the base consumption for *Work* is high on the Thursday already. If a person cannot work on a Thursday, he/she will only be able to allocate a small extra amount of work on a Friday, while he/she might be able to allocate more hours to make up for the time lost during the weekend. This is also true for the discrete part of the model.

As in the 3-day case, we observe that the base model redistributes time more evenly from Thursday to Saturday and Sunday (reflecting correlations, respectively, of 0.71 and 0.61), while in the model with the gender effect, the difference is larger (reflecting correlations, respectively, of 0.93 and 0.17).

4.3.3 Concepción - Forecasting results

The detailed results for the forecasting application to the *Concepción* data are reported in Table 9. We will only comment briefly on those as most of the considerations reported for the *Mobidrive* data also apply in this case. We again subtract $0.5*\mu$ from δ_{WD} for the *Work* activity. We see that the percentage reduction in the share of people performing *Work* during the weekday is similar for all approaches across the model with and without the gender effect, although it is slightly higher in the case of approach B. A higher reduction in the average continuous time allocation to *Work* on the weekday is observed for approach A. The approaches also differ in the pattern of time redistribution across different activities during the weekday. This is again largely a result of the differences in the rates of conducting activities.

As explained above, we see no redistribution to the weekend day for approach A, while the other approaches redistribute time especially to *Work* on the weekend day (in line with the correlations), although there are differences between the model with and without gender. These differences are again a result of different correlation patterns in the two models, this time not in terms of work on a WD and WE, but between WD work and other activities, both on a weekday and weekend.

			Base model				With gender			
			Approach A	Approach B	Approach C1	Approach C2	Approach A	Approach B	Approach C1	Approach C2
Change in share participating	Friday	Outside good	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		Work	**-51.17%**	**-57.51%**	**-54.82%**	**-54.82%**	**-52.45%**	**-58.63%**	**-56.09%**	**-56.09%**
		School	2.72%	0.69%	0.90%	0.90%	2.61%	0.56%	1.03%	1.03%
		Drop-off/ Pick-up	13.46%	6.96%	4.81%	4.81%	14.10%	5.49%	6.05%	6.05%
		Daily shopping	9.73%	4.09%	3.87%	3.87%	10.35%	3.66%	4.21%	4.21%
		Non daily shopping	12.43%	6.06%	4.96%	4.96%	14.26%	6.50%	5.43%	5.43%
		Social	11.46%	4.75%	4.75%	4.75%	12.01%	4.38%	4.63%	4.63%
		Leisure	9.71%	3.22%	3.45%	3.45%	10.14%	4.08%	4.03%	4.03%
		Private business	10.84%	4.51%	4.39%	4.39%	10.35%	3.99%	4.48%	4.48%
		Travel	0.41%	0.24%	0.16%	0.16%	0.41%	0.21%	0.15%	0.15%
	Saturday	Outside good	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		Work	**0.00%**	**14.20%**	**11.33%**	**11.33%**	**0.00%**	**21.79%**	**15.42%**	**15.42%**
		School	0.00%	0.51%	2.50%	2.50%	0.00%	1.62%	2.40%	2.40%
		Drop-off/ Pick-up	0.00%	5.43%	5.51%	5.51%	0.00%	5.35%	5.43%	5.43%
		Daily shopping	0.00%	5.26%	4.67%	4.67%	0.00%	4.91%	4.36%	4.36%
		Non daily shopping	0.00%	4.16%	4.31%	4.31%	0.00%	8.27%	5.99%	5.99%
		Social	0.00%	4.20%	4.02%	4.02%	0.00%	4.40%	4.01%	4.01%
		Leisure	0.00%	4.94%	4.35%	4.35%	0.00%	4.68%	4.76%	4.76%
		Private business	0.00%	4.32%	5.10%	5.10%	0.00%	5.45%	4.29%	4.29%
		Travel	0.00%	1.88%	1.27%	1.27%	0.00%	1.40%	1.10%	1.10%
	Sunday	Outside good	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		Work	**0.00%**	**15.13%**	**11.84%**	**11.84%**	**0.00%**	**6.44%**	**6.75%**	**6.75%**
		School	0.00%	2.05%	1.41%	1.41%	0.00%	1.33%	1.71%	1.71%
		Drop-off/ Pick-up	0.00%	6.33%	5.55%	5.55%	0.00%	3.33%	3.81%	3.81%
		Daily shopping	0.00%	4.70%	4.95%	4.95%	0.00%	4.79%	4.07%	4.07%
		Non daily shopping	0.00%	4.26%	4.82%	4.82%	0.00%	2.38%	2.90%	2.90%
		Social	0.00%	5.85%	5.10%	5.10%	0.00%	5.25%	4.71%	4.71%
		Leisure	0.00%	3.32%	3.59%	3.59%	0.00%	3.73%	3.83%	3.83%
		Private business	0.00%	4.29%	5.04%	5.04%	0.00%	5.12%	5.08%	5.08%
		Travel	0.00%	2.23%	2.04%	2.04%	0.00%	2.12%	2.04%	2.04%
Change in average consumption in sample population	Friday	Outside good	9.48%	7.65%	7.73%	8.01%	9.66%	7.77%	7.92%	8.20%
		Work	**-73.31%**	**-74.12%**	**-75.37%**	**-75.16%**	**-74.70%**	**-75.50%**	**-76.63%**	**-76.43%**
		School	2.11%	0.90%	1.26%	1.31%	2.12%	1.11%	1.44%	1.49%
		Drop-off/ Pick-up	11.06%	8.40%	7.65%	7.89%	11.75%	9.24%	8.40%	8.65%
		Daily shopping	12.72%	9.06%	8.44%	8.72%	13.54%	9.05%	8.95%	9.24%
		Non daily shopping	13.83%	9.88%	9.38%	9.62%	14.27%	9.34%	9.90%	10.16%
		Social	11.22%	6.14%	7.32%	7.52%	12.49%	6.90%	8.13%	8.36%
		Leisure	9.57%	5.58%	6.19%	6.36%	10.24%	6.20%	6.73%	6.91%
		Private business	10.65%	7.96%	7.49%	7.70%	10.48%	7.23%	7.27%	7.47%
		Travel	12.56%	9.75%	10.02%	10.34%	13.20%	10.18%	10.58%	10.92%
	Saturday	Outside good	0.00%	-0.56%	-0.48%	-0.47%	0.00%	-0.56%	-0.51%	-0.51%
		Work	**0.00%**	**12.31%**	**8.88%**	**8.86%**	**0.00%**	**22.81%**	**16.44%**	**16.44%**
		School	0.00%	1.09%	0.82%	0.82%	0.00%	0.84%	0.88%	0.88%
		Drop-off/ Pick-up	0.00%	2.50%	1.72%	1.72%	0.00%	2.57%	1.72%	1.72%
		Daily shopping	0.00%	2.13%	1.64%	1.64%	0.00%	2.28%	1.52%	1.52%
		Non daily shopping	0.00%	2.74%	1.87%	1.87%	0.00%	3.84%	2.68%	2.68%
		Social	0.00%	2.66%	2.43%	2.43%	0.00%	2.31%	2.25%	2.24%
		Leisure	0.00%	2.52%	2.25%	2.24%	0.00%	2.35%	2.04%	2.04%
		Private business	0.00%	1.04%	1.39%	1.39%	0.00%	2.04%	1.60%	1.60%
		Travel	0.00%	0.55%	0.24%	0.25%	0.00%	0.25%	0.16%	0.16%
	Sunday	Outside good	0.00%	-0.50%	-0.40%	-0.38%	0.00%	-0.43%	-0.34%	-0.32%
		Work	**0.00%**	**13.99%**	**9.98%**	**9.94%**	**0.00%**	**3.69%**	**3.60%**	**3.57%**
		School	0.00%	1.21%	1.05%	1.05%	0.00%	2.11%	1.87%	1.86%
		Drop-off/ Pick-up	0.00%	1.92%	1.28%	1.28%	0.00%	1.98%	1.49%	1.49%
		Daily shopping	0.00%	1.60%	1.48%	1.47%	0.00%	0.39%	0.80%	0.80%
		Non daily shopping	0.00%	2.84%	3.05%	3.03%	0.00%	1.45%	0.87%	0.87%
		Social	0.00%	3.88%	3.04%	3.02%	0.00%	3.46%	2.62%	2.60%
		Leisure	0.00%	2.17%	2.01%	2.00%	0.00%	2.72%	2.15%	2.14%
		Private business	0.00%	1.97%	1.36%	1.36%	0.00%	3.55%	2.21%	2.19%
		Travel	0.00%	1.32%	0.72%	0.72%	0.00%	1.22%	0.83%	0.82%

Table 7: Mobidrive - 3 day forecasting approaches

			Base model				**With gender**			
			Approach A	Approach B	Approach C1	Approach C2	Approach A	Approach B	Approach C1	Approach C2
Change in share participating	Thursday	Outside good	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		Work	**-51.13%**	**-63.66%**	**-56.47%**	**-56.47%**	**-52.10%**	**-65.36%**	**-57.90%**	**-57.90%**
		School	2.59%	0.37%	0.68%	0.68%	2.45%	0.33%	0.64%	0.64%
		Drop-off/ Pick-up	13.11%	2.80%	3.33%	3.33%	13.80%	3.81%	3.97%	3.97%
		Daily shopping	9.98%	2.14%	2.62%	2.62%	10.38%	2.45%	2.72%	2.72%
		Non daily shopping	13.58%	3.18%	2.99%	2.99%	14.40%	4.11%	3.77%	3.77%
		Social	11.49%	2.84%	2.79%	2.79%	12.15%	3.16%	3.51%	3.51%
		Leisure	9.19%	1.56%	2.20%	2.20%	10.18%	3.90%	2.09%	2.09%
		Private business	10.78%	1.44%	2.75%	2.75%	10.61%	2.17%	2.74%	2.74%
		Travel	0.37%	0.39%	0.15%	0.15%	0.30%	0.17%	0.13%	0.13%
	Friday	Outside good	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		Work	**0.00%**	**3.01%**	**1.78%**	**1.78%**	**0.00%**	**2.96%**	**1.74%**	**1.74%**
		School	0.00%	0.12%	0.74%	0.74%	0.00%	-0.06%	0.66%	0.66%
		Drop-off/ Pick-up	0.00%	2.39%	2.80%	2.80%	0.00%	1.80%	2.61%	2.61%
		Daily shopping	0.00%	1.67%	2.57%	2.57%	0.00%	1.89%	2.68%	2.68%
		Non daily shopping	0.00%	2.12%	3.72%	3.72%	0.00%	2.05%	3.85%	3.85%
		Social	0.00%	2.58%	2.52%	2.52%	0.00%	3.21%	2.92%	2.92%
		Leisure	0.00%	1.85%	2.61%	2.61%	0.00%	2.28%	2.48%	2.48%
		Private business	0.00%	2.72%	2.80%	2.80%	0.00%	2.40%	2.56%	2.56%
		Travel	0.00%	0.45%	0.11%	0.11%	0.00%	0.27%	0.10%	0.10%
	Saturday	Outside good	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		Work	**0.00%**	**7.48%**	**6.42%**	**6.42%**	**0.00%**	**13.94%**	**9.24%**	**9.24%**
		School	0.00%	0.00%	1.17%	1.17%	0.00%	1.88%	2.11%	2.11%
		Drop-off/ Pick-up	0.00%	2.72%	3.17%	3.17%	0.00%	2.34%	3.00%	3.00%
		Daily shopping	0.00%	2.74%	2.91%	2.91%	0.00%	3.28%	3.02%	3.02%
		Non daily shopping	0.00%	1.91%	2.85%	2.85%	0.00%	4.46%	4.65%	4.65%
		Social	0.00%	2.22%	2.72%	2.72%	0.00%	2.04%	2.82%	2.82%
		Leisure	0.00%	2.01%	3.02%	3.02%	0.00%	1.63%	2.91%	2.91%
		Private business	0.00%	0.89%	2.92%	2.92%	0.00%	2.86%	2.97%	2.97%
		Travel	0.00%	1.27%	0.97%	0.97%	0.00%	0.93%	0.78%	0.78%
	Sunday	Outside good	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		Work	**0.00%**	**10.64%**	**7.52%**	**7.52%**	**0.00%**	**4.49%**	**3.74%**	**3.74%**
		School	0.00%	0.58%	2.01%	2.01%	0.00%	-0.47%	1.39%	1.39%
		Drop-off/ Pick-up	0.00%	3.29%	3.82%	3.82%	0.00%	3.36%	2.28%	2.28%
		Daily shopping	0.00%	2.41%	3.11%	3.11%	0.00%	2.62%	2.98%	2.98%
		Non daily shopping	0.00%	1.24%	2.61%	2.61%	0.00%	0.00%	1.90%	1.90%
		Social	0.00%	3.40%	3.38%	3.38%	0.00%	2.48%	2.96%	2.96%
		Leisure	0.00%	2.46%	2.09%	2.09%	0.00%	2.61%	2.32%	2.32%
		Private business	0.00%	1.70%	2.63%	2.63%	0.00%	2.26%	3.38%	3.38%
		Travel	0.00%	1.47%	1.34%	1.34%	0.00%	1.40%	1.55%	1.55%
Change in average consumption in sample population	Thursday	Outside good	9.61%	7.19%	7.40%	7.67%	9.80%	7.36%	7.57%	7.85%
		Work	**-73.73%**	**-77.02%**	**-76.76%**	**-76.55%**	**-75.03%**	**-78.34%**	**-77.98%**	**-77.77%**
		School	2.17%	0.80%	1.09%	1.13%	2.17%	0.86%	1.18%	1.22%
		Drop-off/ Pick-up	11.68%	7.34%	7.01%	7.26%	12.66%	7.63%	7.74%	8.01%
		Daily shopping	12.32%	7.43%	7.10%	7.34%	13.14%	7.39%	7.52%	7.77%
		Non daily shopping	14.04%	6.85%	8.09%	8.35%	14.92%	8.03%	8.63%	8.91%
		Social	11.40%	5.80%	6.28%	6.47%	12.51%	6.55%	7.05%	7.25%
		Leisure	9.77%	4.19%	5.22%	5.37%	10.37%	5.63%	5.59%	5.74%
		Private business	10.64%	5.20%	6.10%	6.29%	10.30%	5.25%	5.93%	6.13%
		Travel	12.62%	9.29%	9.20%	9.52%	13.19%	9.54%	9.69%	10.02%
	Friday	Outside good	0.00%	-0.27%	-0.19%	-0.20%	0.00%	-0.22%	-0.20%	-0.21%
		Work	**0.00%**	**2.32%**	**1.00%**	**1.00%**	**0.00%**	**1.96%**	**1.04%**	**1.04%**
		School	0.00%	-0.03%	0.20%	0.20%	0.00%	0.01%	0.17%	0.17%
		Drop-off/ Pick-up	0.00%	-0.16%	0.58%	0.58%	0.00%	-0.01%	0.48%	0.48%
		Daily shopping	0.00%	0.15%	0.64%	0.64%	0.00%	0.13%	0.67%	0.67%
		Non daily shopping	0.00%	0.41%	1.10%	1.10%	0.00%	0.46%	1.11%	1.11%
		Social	0.00%	0.97%	0.98%	0.99%	0.00%	1.43%	1.06%	1.06%
		Leisure	0.00%	0.88%	0.90%	0.91%	0.00%	1.07%	1.02%	1.02%
		Private business	0.00%	0.80%	0.70%	0.70%	0.00%	0.22%	0.58%	0.58%
		Travel	0.00%	-0.50%	-0.06%	-0.06%	0.00%	-0.64%	-0.07%	-0.08%
	Saturday	Outside good	0.00%	-0.29%	-0.28%	-0.27%	0.00%	-0.30%	-0.28%	-0.27%
		Work	**0.00%**	**8.41%**	**5.18%**	**5.15%**	**0.00%**	**14.83%**	**9.06%**	**9.05%**
		School	0.00%	0.17%	0.34%	0.33%	0.00%	1.23%	0.85%	0.85%
		Drop-off/ Pick-up	0.00%	2.04%	1.08%	1.07%	0.00%	0.68%	0.91%	0.90%
		Daily shopping	0.00%	0.63%	1.14%	1.13%	0.00%	0.98%	1.26%	1.26%
		Non daily shopping	0.00%	1.32%	1.32%	1.31%	0.00%	3.19%	1.90%	1.89%
		Social	0.00%	1.56%	1.70%	1.69%	0.00%	1.41%	1.53%	1.52%
		Leisure	0.00%	1.47%	1.45%	1.44%	0.00%	1.12%	1.24%	1.23%
		Private business	0.00%	0.00%	0.95%	0.94%	0.00%	0.82%	0.85%	0.84%
		Travel	0.00%	0.52%	0.24%	0.25%	0.00%	0.51%	0.19%	0.19%
	Sunday	Outside good	0.00%	-0.25%	-0.23%	-0.22%	0.00%	-0.21%	-0.20%	-0.19%
		Work	**0.00%**	**11.57%**	**6.84%**	**6.80%**	**0.00%**	**4.55%**	**2.48%**	**2.45%**
		School	0.00%	0.40%	0.88%	0.87%	0.00%	0.02%	1.81%	1.81%
		Drop-off/ Pick-up	0.00%	1.51%	0.83%	0.82%	0.00%	1.27%	0.80%	0.79%
		Daily shopping	0.00%	-0.61%	1.19%	1.18%	0.00%	0.53%	0.66%	0.65%
		Non daily shopping	0.00%	1.89%	1.82%	1.80%	0.00%	-1.81%	0.63%	0.62%
		Social	0.00%	2.43%	2.02%	2.00%	0.00%	1.86%	1.75%	1.73%
		Leisure	0.00%	1.55%	1.32%	1.31%	0.00%	1.65%	1.39%	1.37%
		Private business	0.00%	0.35%	0.77%	0.77%	0.00%	0.51%	1.43%	1.43%
		Travel	0.00%	0.54%	0.53%	0.53%	0.00%	0.74%	0.59%	0.59%

Table 8: Mobidrive - 4 day forecasting approaches

			Base model				**With gender**			
			Approach A	Approach B	Approach C1	Approach C2	Approach A	Approach B	Approach C1	Approach C2
Change in share participating	Weekday	Outside good	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		Work	**-60.58%**	**-65.97%**	**-64.36%**	**-64.36%**	**-60.86%**	**-66.92%**	**-65.32%**	**-65.32%**
		Drop-off/ Pick-up	18.19%	8.04%	10.27%	10.27%	18.62%	10.12%	10.01%	10.01%
		Social	12.74%	5.83%	6.69%	6.69%	12.20%	5.72%	6.42%	6.42%
		Travel	1.80%	1.12%	1.18%	1.18%	1.47%	0.85%	0.90%	0.90%
		Family	19.91%	9.77%	10.77%	10.77%	18.82%	8.92%	10.12%	10.12%
		Household obligations	14.20%	7.29%	7.47%	7.47%	15.26%	8.07%	8.21%	8.21%
		Out-of-home recreation	12.99%	6.11%	6.92%	6.92%	12.18%	5.29%	5.80%	5.80%
		In-home recreation	14.53%	6.74%	7.13%	7.13%	14.85%	5.57%	7.73%	7.73%
		Services	16.17%	7.54%	8.38%	8.38%	16.25%	7.30%	8.32%	8.32%
		Shopping	13.47%	6.75%	7.72%	7.72%	14.09%	6.63%	7.35%	7.35%
		Study	12.64%	6.54%	6.69%	6.69%	12.05%	5.37%	6.32%	6.32%
	Weekend	Outside good	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		Work	**0.00%**	**9.19%**	**9.82%**	**9.82%**	**0.00%**	**9.41%**	**9.93%**	**9.93%**
		Drop-off/ Pick-up	0.00%	9.91%	9.66%	9.66%	0.00%	8.81%	10.98%	10.98%
		Social	0.00%	5.99%	6.18%	6.18%	0.00%	5.36%	6.11%	6.11%
		Travel	0.00%	3.78%	3.80%	3.80%	0.00%	3.03%	3.29%	3.29%
		Family	0.00%	8.24%	7.87%	7.87%	0.00%	7.00%	6.98%	6.98%
		Household obligations	0.00%	6.92%	8.36%	8.36%	0.00%	7.98%	8.28%	8.28%
		Out-of-home recreation	0.00%	6.95%	7.27%	7.27%	0.00%	6.61%	7.27%	7.27%
		In-home recreation	0.00%	6.54%	7.45%	7.45%	0.00%	4.42%	5.62%	5.62%
		Services	0.00%	6.65%	8.33%	8.33%	0.00%	6.65%	9.25%	9.25%
		Shopping	0.00%	8.12%	8.79%	8.79%	0.00%	7.56%	8.40%	8.40%
		Study	0.00%	5.76%	5.96%	5.96%	0.00%	4.44%	6.08%	6.08%
Change in average consumption in sample population	Weekday	Outside good	10.96%	4.40%	0.34%	-5.10%	11.10%	3.29%	-0.96%	-6.35%
		Work	**-67.88%**	**-60.19%**	**-59.15%**	**-60.03%**	**-68.65%**	**-61.98%**	**-60.92%**	**-61.77%**
		Drop-off/ Pick-up	20.01%	14.54%	30.70%	26.48%	21.46%	14.75%	29.90%	25.76%
		Social	14.25%	31.37%	38.68%	35.12%	14.56%	27.65%	34.80%	31.18%
		Travel	13.51%	7.52%	22.90%	18.88%	13.84%	5.26%	21.02%	17.04%
		Family	19.70%	32.52%	42.92%	39.31%	20.45%	38.87%	46.02%	42.56%
		Household obligations	17.06%	37.18%	43.45%	40.03%	18.07%	41.15%	46.53%	43.14%
		Out-of-home recreation	13.16%	26.54%	34.73%	31.03%	12.02%	13.74%	24.27%	20.40%
		In-home recreation	13.53%	33.84%	41.19%	37.66%	14.48%	41.89%	45.92%	42.56%
		Services	17.40%	27.17%	37.92%	34.12%	16.74%	23.02%	34.94%	31.12%
		Shopping	15.77%	15.40%	27.29%	23.37%	15.87%	13.96%	25.91%	21.98%
		Study	13.42%	36.03%	41.76%	38.31%	11.72%	32.78%	38.29%	34.86%
	Weekend	Outside good	0.00%	-9.87%	-12.62%	-7.56%	0.00%	-9.10%	-11.88%	-6.80%
		Work	**0.00%**	**35.60%**	**38.90%**	**42.17%**	**0.00%**	**45.14%**	**46.18%**	**49.19%**
		Drop-off/ Pick-up	0.00%	2.00%	15.29%	19.21%	0.00%	-2.38%	12.61%	16.53%
		Social	0.00%	17.38%	24.34%	27.92%	0.00%	21.43%	28.28%	31.77%
		Travel	0.00%	0.01%	12.17%	15.98%	0.00%	-1.05%	12.03%	15.86%
		Family	0.00%	32.56%	36.30%	39.60%	0.00%	19.97%	26.40%	29.95%
		Household obligations	0.00%	31.58%	34.73%	37.89%	0.00%	35.72%	37.58%	40.69%
		Out-of-home recreation	0.00%	21.14%	28.04%	31.53%	0.00%	21.95%	28.99%	32.49%
		In-home recreation	0.00%	30.36%	34.33%	37.51%	0.00%	27.40%	31.13%	34.39%
		Services	0.00%	26.96%	31.74%	35.21%	0.00%	22.61%	30.54%	34.04%
		Shopping	0.00%	6.72%	16.55%	20.29%	0.00%	10.58%	19.61%	23.29%
		Study	0.00%	25.91%	31.05%	34.35%	0.00%	16.19%	24.84%	28.39%

Table 9: Concepción - forecasting approaches

5 Conclusions

The MDCEV modelling framework has established itself as a preferred method for modelling time allocation, with data very often coming from travel or activity diaries. However, while many of these datasets contain information on multiple days for the same individual, the standard modelling approach has treated each day in isolation. This paper has made the case that not only does this miss out on important links between days, but it potentially leads to issues also in forecasting.

We started by discussing possible ways of accommodating links across days within an MDCEV framework. While the implementation of a non-additive utility function would be the theoretically correct way to accommodate the complementarities and substitutions across days, such an approach is very difficult to estimate and apply in practice, especially with budget constraints at the day and multi-day level. We instead rely on additive utility functions where we accommodate correlation between activities at the within-day and between-day level. This can be accommodated through a mixed MDCEV model, with multi-variate random distributions.

While the use of a mixed MDCEV model in this manner allows us to capture correlations across days without the use of non-additive utility functions and by relying on a simple day-level budget, it raises the issue of how to allow for substitution across days in model application, i.e. forecasting. We discuss how the standard Pinjari and Bhat (2010) approach will fail to make this link and instead propose two different adaptations of this algorithm that relax the 24 hour constraint in forecasting and then reinstate it through rescaling.

We illustrate the issue and the methods using two different datasets, a 2-day activity diary from *Concepción* (Chile) and two weeks of the six weeks *Mobidrive* study (Germany). Our estimation work confirms the presence of deterministic and random heterogeneity, and crucially in the context of the present paper, correlations both at the within-day and between-day level. We then test the proposed forecasting approaches, showing that they lead to more behaviourally meaningful results than the simple day-level forecasts. We also show that the loss in terms of utility of these "alternative" approaches is relatively small when compared to the theoretically correct model. The fact that we obtain consistent insights from two different datasets adds further empirical weight to our paper.

As always, there is substantial scope for further work, both in terms of refined model specification at the empirical end (e.g. other distributions and more covariates) and in terms of further theoretical work, be it with other forecasting approaches or the incorporation of substitution between days at the model level. We believe that the work conducted here is a first step in this direction and further empirical testing of our approaches on other data is thus welcome too.

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