

# The value of travel time: random utility versus random valuation

Manuel Ojeda-Cabral<sup>a\*</sup>

Richard Batley<sup>a</sup>

Stephane Hess<sup>a</sup>

<sup>a</sup> *Institute for Transport Studies, University of Leeds*

\* *Tel: +44 (0)7834 738 505; Email: [M.A.OjedaCabral@leeds.ac.uk](mailto:M.A.OjedaCabral@leeds.ac.uk)  
30-40 University Road, Leeds LS2 9JT*

*Accepted for Publication*

*Transportmetrica A: Transport Science*

# **The value of travel time: random utility versus random valuation**

## **Abstract**

This paper identifies, relates and compares two popular modelling approaches to estimate the value of travel time changes. The first (random utility) assumes that the random component of the model relates to the difference between the utilities of travel options; the second (random valuation) assumes that it relates to the difference between the value of travel time and a suggested valuation threshold. This paper gives details of the theoretical relationship between the two approaches and compares them empirically at several levels of model sophistication. Datasets from two national studies (UK and Denmark) are employed. The results show a consistent superiority of the random valuation approach and a systematic gap in the value of travel time between approaches. A similar pattern across models is found in both countries. This raises questions about the validity of results using the random utility approach. The analysis has direct implications for both researchers and policy-makers.

**Keywords:** value of travel time; discrete choice modelling; stated choice; random utility

## 1 Introduction

The value of travel time changes (VTTC) is a monetary measure of the value that people place on changes in the travel time of their journeys. The VTTC is a key input for the evaluation and comparison of different transport projects. Travel time savings often constitute a major part of the benefits of a project, and therefore the value assigned to them is crucial for cost-benefit analyses (De Rus and Nash, 1997; Wardman, 1998). Several countries conduct national studies to estimate an official VTTC that can be used for appraisal of transport projects. These VTTC estimates can determine which projects are selected by the policy maker, and can therefore shape transport policy and planning. Unfortunately, the VTTC is a subtle concept that cannot be observed directly. The general agreement is that an individual's travel choices that involve trading off travel time changes against changes in travel cost can provide researchers with an approximation to the underlying VTTC of the individual. To make things harder, the VTTC varies across individuals and travel choice contexts.

The theory of the VTTC has been well rehearsed since the seminal work by Becker (1965) and DeSerpa (1971). See Small (2012) for a recent review. In practice, Stated Choice (SC) experiments are typically employed to collect data on travellers' choices that involve a time-cost trade-off. Many SC experiments, including a majority of national studies in Europe, have used a very simple design: respondents are presented with hypothetical choice scenarios that contain two travel alternatives that differ only in terms of travel time and travel cost (i.e. a time-cost trade-off). This has been the case in the UK (Mackie et al., 2003), The Netherlands (HCG, 1998), Denmark (Fosgerau et al., 2007), Norway (Ramjerdi et al., 2010) and Sweden (Börjesson and Eliasson, 2014). This kind of (SC) data is then analysed using discrete choice models to estimate the VTTC.

Multinomial logit models and more recently, thanks to the advances in econometrics, mixed logit models have been commonly used to estimate the VTTC (Börjesson and Eliasson, 2014). From this point, the existing literature starts to be unclear. There is a lack of clarity in the definition and classification of the main modelling approaches used on datasets of the type described above. The first objective of this paper is to make clear what the main modelling approaches are, avoiding confusing definitions or descriptions.

To begin with, there are parametric and non-parametric estimation techniques. Only parametric models are considered in this paper (non-parametric techniques are useful, as they allow the estimation of the statistical distribution of the VTTC, but only as a complement). The more informative parametric models allow the VTTC to vary with covariates, which seems essential and is highly recommended (Börjesson and Eliasson, 2014). The traditional parametric approach is the Random Utility (RU) model. However, in the context of official national VTTC studies using binary time/money trade-offs, two main parametric approaches are identified: the first (Random Utility) assumes that the random component of the model relates to the difference between the utilities of travel options, the second (Random Valuation) assumes that it relates to the difference between the actual value of travel time and a suggested valuation threshold (implicit in the changes in time and cost offered). Both approaches are equivalent in a deterministic domain and can be derived from standard microeconomic theory.

The theoretical relationship between the two modelling approaches is known (Fosgerau et al., 2007; Börjesson and Eliasson, 2014; Hultkranz et al., 1996), although we are not aware of any work which formally shows its derivation fully and therefore clarification is needed. It is also known that the choice of approach is an empirical matter (Börjesson and Eliasson,

2014; Fosgerau, 2007). However, the few published studies that acknowledge both approaches merely state the superiority of the approach they select, without showing any comparative results. Only Hultkrantz et al. (1996) offer some comparative results in an unpublished working paper. Also, the RV approach has always been presented using a logarithmic transformation (Fosgerau, 2007), whereas the numerous applications of the RU approach use, in many cases, a linear specification (Fosgerau and Bierlaire, 2009). Finally, it must be mentioned that the two approaches can also be identified within the series of model transformations tested by Daly and Tsang (2009), which provide additional empirical evidence. Overall, we believe that a rigorous empirical comparison of the two approaches is needed, given the extensive use of the RU model in the field (Daly et al., 2014) and the emergence of the RV model. The VTTC can have large implications for the evaluation of transport policies and infrastructure investments, and thus is necessary to understand whether these implications differ depending on whether RU or RV is used. It is notable that none of the previous papers show a direct empirical comparison of the two models.

In this paper, the rationale of the two approaches is clarified, inspired by Cameron and James (1987)'s original exposition. This is followed by the exposition of the full derivation of the theoretical relationship between the two. The main contribution of the paper is the empirical comparison of the two approaches at several levels of model sophistication. These levels include: i) base linear specification, ii) base logarithmic specification, iii) observed heterogeneity, and iv) random heterogeneity. This procedure ensures fairness in the comparison and allows us to disentangle the real differences between the two approaches, and among levels of sophistication, in terms of VTTC estimates and model fit.

The models are estimated on two datasets corresponding to the national VTTC studies in the UK and Denmark. Since both datasets were obtained using the same SC design, this is also a unique opportunity to observe potential differences across countries. Note that surveys on VTTC using this type of SC design are widely used in European studies and also in some international toll road studies, and our work is thus particularly important in this context.

## **2 The 'problem' of the data collection**

The main problem is that the VTTC cannot be observed directly. In most markets, prices serve as indicators of consumers' valuation of the good. Here, the good analysed is *one minute of travel time*. Changes in travel time are bundles (of different sizes, e.g. 5, 10 or 20 minutes) of this good. However, there is not an obvious market for this good: only travel choice contexts between fast-and-expensive *versus* slow-and-cheap options resemble a market. The implicit time-cost tradeoff would be the 'price' of the good. The VTTC is conceptualized as a measure of how much money (monetary travel cost) a person is willing to exchange for one minute of travel time.

The most popular way to collect information about the VTTC is through SC experiments. It is common, especially in Europe, to find SC experiments that simply offer individuals two travel alternatives that differ only in time and cost. This is equivalent to say that a 'price' is offered to respondents at which they can buy or sell the good (time). Normally, several 'prices' are offered to each individual in separated choice scenarios (often around 8 or 9). In each scenario, individuals choose whether to accept the offered price or not. Hence, they reveal whether, in that context, their VTTC is below or above the offered price. In a more general valuation context, these relatively simple SC experiments are known as 'referendum surveys' or 'closed-ended contingent valuation surveys' (Cameron and James,

1987). Compared to more complex SC experiments where additional attributes and/or alternatives are included, a key feature of referendum surveys is that they have directly observable threshold levels for the unobservable variable of interest (in this case the VTTC): i.e. referendum surveys mimic a market offering a price.

Most national VTTC studies use this kind of SC experiment, including the most recent UK and Danish studies.

### ***2.1 A common Stated Choice design***

Given the hypothetical nature of SC experiments, it is common to relate choice scenarios to respondents' previous travel experiences. In the UK and Danish national studies, respondents were recruited while travelling and information about a recent trip was collected. This trip, defined by current travel time  $T$  and current travel cost  $C$ , is used as the reference trip throughout the survey. The participants are then presented with eight choice scenarios, each with two travel options ( $i=1,2$ ) varying in cost ( $c_i$ ) and time ( $t_i$ ) with values around the reference trip. One option is always faster but more expensive. Data can always be reordered to give an option 1 that is cheaper but slower than option 2 (i.e.  $t_1 > t_2$  and  $c_1 < c_2$ ). A special characteristic of the SC designs in several of these European national studies, including the most recent UK and Danish ones, is that  $T$  and  $C$  always coincide with one of the time and cost levels. Therefore, travellers are always considering a given change in time ( $\Delta t_i = t_i - T$ ) against a given change in cost ( $\Delta c_i = c_i - C$ ). Those changes coincide, under this setting, with the differences in time and cost between the alternatives. In short, there is always an implicit 'price' which is called the boundary VTTC (BVTTC). The BVTTC is defined as:

$$BVTTC = \frac{-(c_2 - c_1)}{(t_2 - t_1)} = -\frac{\Delta c}{\Delta t} \quad (1)$$

It should be noted that the same boundary valuations can of course also be calculated in other binary time/money trade-offs where neither of the alternatives uses the reference time or cost values.

### ***2.2 Datasets in the UK and Denmark***

The information presented so far constitutes the essence of the SC design, common for the most recent UK and Danish studies. The range of values employed and the presentation of the scenarios differ according to the specific circumstances of each study. At the end of the survey, once each respondent had completed the eight scenarios, information about several socio-demographics was also collected in both studies. For reasons of comparability and homogeneity, this paper focuses on car travel for non-business (i.e. commute and other non-business travel) purposes. Only small differences exist in these datasets between commute and other non-business travel, and they are not relevant for the purpose of our analysis. In particular, only drivers' responses are used (i.e. no use is made of passengers' responses available only in the UK data).

The dataset employed in the UK was collected in 1994 by Accent and Hague Consulting Group (AHCG, 1996) using paper questionnaires. It contains 10,598 valid observations of individuals' choices from 1,565 respondents. The re-analysis by Mackie et al. (2003) led to the establishment of the current VTTC values officially employed in the UK.

The dataset employed for the Danish Value of Time study (DATIV) was collected in 2004. It was designed by RAND Europe (Burge et al., 2004) and analysed by Fosgerau et al. (2007) to update the official VTTC in Denmark. Travellers were interviewed online or face-to-face through computer assisted personal interview. It contains 17,020 observations from 2,197 respondents.

Travel time is expressed in minutes in both countries, while travel cost is shown in pence in the UK and in Danish Kroner (DKK) in Denmark. The selected attribute levels and consequent boundary VTTC cover the ranges displayed in table 1.

Table 1. Stated Choice design variables

Design variable	Minimum level		Maximum level	
	UK	Denmark	UK	Denmark
$\Delta t_i$ (in minutes)	-20	-60	+20	+60
$\Delta c_i$	-300 pence	-200 DKK	+300 pence	+175 DKK
Boundary VTTC	1 pence/minute	2 DKK/hour	25 pence/minute	200 DKK/hour

It is interesting to see that the range for the changes in travel time and travel cost is much broader in Denmark, while the maximum boundary VTTC levels are rather similar (considering an exchange rate of  $1\text{£} \approx 9\text{DKK}$ ). This shows that none of the studies focused particularly on the right tail of the VTTC distribution.

Both studies differed also in the way choice scenarios were presented. While in the UK the values of  $\Delta c_i$  and  $\Delta t_i$  were displayed under each travel option, the Danish respondents are presented with the final levels of cost ( $c_i$ ) and time ( $t_i$ ), made possible by the computer based presentation. For example, given  $T=20$  and  $C=100$ , the same scenario would be presented respectively as indicated in table 2.

Table 2. Presentation of Stated Choice scenarios

Attribute	UK		Denmark	
	Option A	Option B	Option A	Option B
Time	As now	10 minutes shorter than now	20	10
Cost	As now	50 pence higher than now	100 DKK	150 DKK

### 3 Rationale of two modelling approaches for the VTTC

Two relevant modelling approaches are identified in the literature. Both are well rooted in microeconomic theory (they are equivalent in a deterministic domain) and use discrete choice models to analyse choices from SC experiments. A microeconomic consumption problem, where individuals are assumed to make choices in order to maximise their utility, is the starting point (e.g. Becker, 1965; DeSerpa, 1971; Jara Diaz, 2003). From the microeconomic problem, a conditional indirect utility function  $V_i$  is derived.  $V_i$  is the key element of the discrete choice models. Together with  $V_i$ , an error term  $\varepsilon_i$  that accounts for unobserved factors is also necessary to enter the stochastic world of econometrics. There are differing interpretations of the error term in the literature (e.g. Block and Marschak, 1960; McFadden, 1976; Train, 2009), but broadly speaking  $\varepsilon_i$  would account for any inter-individual and intra-individual variation in preference orderings that is unobservable to the researcher.

#### 3.1 Random Utility (RU) approach

For a long time, it has been standard to define utility ( $U_i$ ) as an observable measure of the attractiveness of each travel alternative ( $V_i$ ) plus an error term ( $\varepsilon_i$ ) assumed to follow a Gumbel distribution (type-I generalized extreme value (GEV) distribution) with constant variance (Daly et al., 2014). The attractiveness of each alternative is represented by its main attributes (time and cost in this case):

$$U_i = (V_i, \varepsilon_i) = \beta_c c_i + \beta_t t_i + \varepsilon_i \quad (2)$$

Where  $\beta_t$  and  $\beta_c$  are the marginal utilities of time and cost respectively. Then the differences in utility between travel alternatives drive people's choices ( $y$ ):

$$y = 1\{\beta_c c_1 + \beta_t t_1 > \beta_c c_2 + \beta_t t_2 + \varepsilon\} \quad (3)$$

The VTTC is obtained as the ratio between time and cost marginal utilities, i.e. the ratio of the partial derivatives of the utility against travel time and cost:

$$VTTC = \frac{\beta_t}{\beta_c} \quad (4)$$

Adding an i.i.d extreme value error term to ' $\beta_c c_i + \beta_t t_i$ ' implies that the difference in the attractiveness of each travel option is distributed with a constant variance across observations.

This approach is known as the Random Utility Model (RUM) and has been widely used since McFadden's (1974) seminal work and Daly and Zachary's (1975) work in the VTTC context. However, there are other options. If the utility function  $V_i$  is derived from microeconomic theory (see e.g. Train and McFadden, 1978; Jara Diaz, 2002), it is only

required to be modelled as a function of the levels of cost and time ( $c_i$  and  $t_i$ ) of the  $i$  options considered by the decision-maker. This is:

$$V_i = f(c_i, t_i) \quad (5)$$

But microeconomic theory does not state anything regarding the introduction of the error term, which is purely empirical issue. The introduction of the error in line with equation (2) is just one option. In other words, another element of the model could, in principle, be assumed to be distributed with constant variance. However, the tendency to think in terms of ‘utility’ and the complexity of many choice scenarios made the RU approach standard for many years. The ‘automatic’ thinking in terms of ‘travel options’ and the utilities associated with them has arguably been restrictive, and may have been the source of misunderstandings and biases on VTTC estimation.

### 3.2 Random Valuation (RV) approach

Cameron and James (1987) realised this and suggested an alternative approach, feasible with a particular type of data. Referendum data (employed in most VTTC national studies) are different from typical discrete choice data (Cameron, 1988), and arguably facilitates simpler interpretations of the stated choices. The rationale for the RV approach is hence related to the existence of referendum data.

Having only two travel options differing in time and cost (i.e. referendum data), a price of *one minute of travel time* is implicit and is observable (i.e. the BVTTC). Therefore, people’s travel choices can be rationalized as part of a hypothetical ‘time market’, where they directly accept or reject the price offered based on their valuation of the good. One can alternatively see the choice options as ‘*buying time*’ and ‘*not buying time*’ at a given price. If the objective is the VTTC, this is a more direct approach than thinking about ‘random utility’ in the sense of equation (2), and is possible because a threshold price is observable.

The individual can therefore decide whether: 1) to buy time, in which case a VTTC equal or greater than the price is revealed; 2) not to buy time, revealing a VTTC lower than the price. The individuals’ choice probabilities will be driven by the difference between the true VTTC and the BVTTC:

$$y = 1\{VTTC < BVTTC + \varepsilon\} \quad (6)$$

Adding an i.i.d extreme value error term to the VTTC and the BVTTC implies that the difference between valuation and price is distributed with a constant variance across observations, which is a reasonable alternative to the RU approach. This is the essence of the approaches described by Cameron and James (1987), Cameron (1988) and more recently by Fosgerau et al. (2007), being implemented for the most recent Danish, Norwegian and Swedish national VTTC studies.

With the existing methodology, this dichotomy between approaches has only been developed in a binary choice context with two attributes. It is clear that ‘buying time’ is equivalent to choosing the fast option, and ‘not buying time’ is equivalent to choosing the



slowest option. Hence both ways of approaching the decision-making process are totally equivalent in a deterministic context. It is the two different assumptions on the inclusion of the error terms what give place to two econometric approaches.

### 3.3 Terminology

Confusion exists around how to distinguish the approaches with adequate terminology. This paper uses the terminology employed by Hultkrantz et al. (1996), who name the latter (equation 6) the Random Valuation (RV) approach. Nevertheless, *utilities* are associated with *options*, and *options* can be rationalized in different ways. Hence, a model based on equation (6) could still be rationalized as a Random Utility (RU) model (e.g. Fosgerau et al. (2007) define VTTC and BVTTC as ‘pseudo-utilities’). Similarly, some particular type of RU in the sense of equation (2) would include random valuation (e.g. mixed logit model). On the other hand, Börjesson and Eliasson (2014) define them as ‘Estimating in Marginal Utility (MU) space’ and ‘Estimating in Marginal Rate of Substitution (MRS) space’ respectively. This also seems confusing, as models based on marginal utilities (also known as ‘preference space’) are often transformed to estimate in MRS-space (also known as ‘willingness-to-pay space’) but without changing the error term structure. As a consequence (continuing with the confusing terminology), the typical specification for a RV model, which is in logarithms (i.e. log RV), has also been referred to as log-WTP model (e.g. Börjesson and Eliasson, 2014). Having noted the potential confusions, Hultkrantz et al.’s (1996) terminology is still for us the most accurate one and is employed throughout the present paper.

## 4 Theoretical relationship between two approaches

The theoretical relationship between the two modelling approaches is known (Börjesson and Eliasson, 2014; Hultkrantz et al., 1996; Fosgerau, 2007), but we are not aware of any work which formally shows its derivation in the stochastic domain step by step.

### 4.1 Deterministic domain

The observable part of the utilities can be defined according to the RU approach as:

$$\begin{cases} V_1 = \beta_t * t_1 + \beta_c * c_1 \\ V_2 = \beta_t * t_2 + \beta_c * c_2 \end{cases} \quad (7)$$

And according to the RV approach as follows:

$$\begin{cases} V_1 = \text{BVTTC} = \frac{-(c_2 - c_1)}{(t_2 - t_1)} \\ V_2 = \text{VTTC} = \frac{\beta_t}{\beta_c} \end{cases} \quad (8)$$

The equivalence between equations 7 and 8 in terms of individuals' choices can be formally shown as follows (e.g. Fosgerau et al., 2007). If the slow option 1 is chosen, then the VTTC is lower than the BVTTC:

$$\beta_t * t_1 + \beta_c * c_1 > \beta_t * t_2 + \beta_c * c_2 \quad (9)$$

$$\beta_t * (t_1 - t_2) > -\beta_c * (c_1 - c_2) \quad (10)$$

$$\frac{\beta_t}{\beta_c} < -\frac{(c_1 - c_2)}{(t_1 - t_2)} \quad (11)$$

#### 4.2 Stochastic domain

The difference between the two approaches lie in the way randomness is introduced (Fosgerau et al., 2007). The most common procedure is to add an extreme value error term to  $V_i$ . As Hultkrantz et al. (1996) reflect, the key question is which element of the choice problem is distributed with a constant variance, (or, similarly, which element is used to define the observable utility function):

- a) a measure of the attractiveness of a travel option, i.e.  $\beta_c c_i + \beta_t t_i$  or
- b) the VTTC.

The utility functions in equations (7) and (8) need to be extended for estimation. An additive error term is added, leading to RU and RV approaches respectively. The errors in each model have different implications and hence different notation is employed:

$$\begin{cases} \widetilde{U}_1 = \widehat{\beta}_t * t_1 + \widehat{\beta}_c * c_1 + \widetilde{\varepsilon}_1 \\ \widetilde{U}_2 = \widehat{\beta}_t * t_2 + \widehat{\beta}_c * c_2 + \widetilde{\varepsilon}_2 \end{cases} \quad (12)$$

$$\begin{cases} U_1 = \mu * \text{BVTTC} + \varepsilon_1 \\ U_2 = \mu * \text{VTTC} + \varepsilon_2 \end{cases} \quad (13)$$

Where the errors ( $\varepsilon_i$ ) are i.i.d,  $\mu$  is a scale parameter,  $\widehat{\beta}_t = \mu\beta_t$  and  $\widehat{\beta}_c = \mu\beta_c$  ( $\mu$  cannot be identified in equation (12) separately from the marginal utilities). In order to show the theoretical relationship, equation (13) will be related to equation (12) through a series of transformations.

Equation (13) can be rearranged to obtain:

$$\begin{cases} U_1 = 0 + \varepsilon_1 \\ U_2 = \mu * \text{VTTC} - \mu * \text{BVTTC} + \varepsilon_2 \end{cases} \quad (14)$$

Multiplying (14) by the marginal utility of cost  $\beta_c$ :

$$\begin{cases} \beta_c U_1 = 0 + \beta_c * \varepsilon_1 \\ \beta_c U_2 = \beta_c * \mu * VTTC - \beta_c * \mu * BVTTC + \beta_c * \varepsilon_2 \end{cases} \quad (15)$$

Multiplying (15) by the change in travel time ( $\Delta t$ ) offered:

$$\begin{cases} \Delta t \beta_c U_1 = 0 + \Delta t * \beta_c * \varepsilon_1 \\ \Delta t \beta_c U_2 = \Delta t * \beta_c * \mu * VTTC - \Delta t * \beta_c * \mu * BVTTC + \Delta t * \beta_c * \varepsilon_2 \end{cases} \quad (16)$$

Substituting in (16) based on the definition of the VTTC (4) and BVTTC (1):

$$\begin{cases} \Delta t \beta_c U_1 = 0 + \Delta t * \beta_c * \varepsilon_1 \\ \Delta t \beta_c U_2 = \mu * \beta_t * \Delta t + \mu * \beta_c * \Delta c + \Delta t * \beta_c * \varepsilon_2 \end{cases} \quad (17)$$

Equation (17) can be written as:

$$\begin{cases} \widetilde{U}_1 = 0 + \widetilde{\varepsilon}_1 \\ \widetilde{U}_2 = \widehat{\beta}_t * \Delta t + \widehat{\beta}_c * \Delta c + \widetilde{\varepsilon}_2 \end{cases} \quad (18)$$

where:

$$\begin{aligned} \widetilde{\varepsilon}_i &= \Delta t * \beta_c * \varepsilon_i \\ \widetilde{U}_i &= \Delta t \beta_c U_i \\ \widehat{\beta}_t &= \mu * \beta_t \\ \widehat{\beta}_c &= \mu * \beta_c \\ \Delta t &= (t_2 - t_1) \\ \Delta c &= (c_2 - c_1) \end{aligned}$$

The utilities in (18) resemble those of model (12). The relationship between the approaches is summarized in the following expression:  $\widetilde{\varepsilon}_i = \Delta t * \beta_c * \varepsilon_i$ . Both approaches can be interpreted as variant of the other but with a particular form of heteroskedastic errors (Börjesson and Eliasson, 2014). If the VTTC has in fact constant variance (*RV approach*), defining the model in line with *RU approach* would cause the error terms to be heteroskedastic with their variance being proportional to the change in travel time (Hultkranz et al., 1996).

Which approach is the best representation of reality is an empirical matter. Existing evidence suggest the RV approach explains choices better (Börjesson and Eliasson, 2014;

Fosgerau, 2007; Börjesson et al., 2012). Nonetheless, there is very limited evidence and the comparison between approaches has been made in a different way. Fosgerau (2007) uses non-parametric techniques to observe which model would be more consistent with the data before making any modelling assumptions. Interestingly, (to the best of our knowledge) all existing empirical works using the RV approach consider a logarithmic extension of the model, while this is not the case for most works based on the RU approach. The only works reporting some comparative results using parametric techniques are: i) an unpublished working paper by Hultkrantz et al. (1996), where it is shown that there may be substantive differences in the VTTC estimation from both approaches; ii) a paper by Daly and Tsang (2009) in which they explore impacts of different transformations and scaling of utility functions, among which we could identify the specifications that would correspond to the RU and RV approaches. However, it remains unclear the extent to which any differences in VTTC hinted at in existing papers actually accrue solely to the selection of a RU or RV model. Providing a rigorous comparison of the two approaches seems necessary.

## 5 Empirical work: comparing the two approaches

In this section the two approaches are compared empirically. The comparison is carried out at several levels of model sophistication. The two base linear models in equations (12) and (13) are incrementally extended. The objective is to investigate:

- i) The difference in the VTTC and model fit between the approaches after subsequent identical modifications: additive error terms (linear base), multiplicative error terms (logarithmic base), observed heterogeneity and random heterogeneity.
- ii) The impact of each model extension on the VTTC and model fit separately within each approach.

### 5.1 Model specification 1: Linear base models (additive error terms)

The first level of comparison is the linear base models described in the previous section. However, to make the comparison more straightforward, *the RU approach* will be expressed in terms of the VTTC (this needs a rearrangement of equation (12) which does not affect any of the results). Additionally, to simplify notation the error terms are introduced in each model using the same Greek letter *epsilon*. The relationship between the two models should be kept in mind as explained in the previous section.

#### RU approach

$$\begin{cases} U_1 = \beta_c \left( \frac{\beta_t}{\beta_c} * t_1 + c_1 \right) + \varepsilon_1 \\ U_2 = \beta_c \left( \frac{\beta_t}{\beta_c} * t_2 + c_2 \right) + \varepsilon_2 \end{cases} \quad (19)$$

#### RV approach

$$\begin{cases} U_1 = \mu * \text{BVTTC} + \varepsilon_1 \\ U_2 = \mu * \text{VTTC} + \varepsilon_2 \end{cases} \quad (20)$$

With the VTTC defined as:

$$\text{VTTC} = \frac{\beta_t}{\beta_c} = \beta_0 \quad (21)$$

Where  $\beta_0$  is a parameter to be estimated. Both models are defined in VTTC space, where  $\beta_0$  is used to represent the main coefficient for the VTTC.

Throughout the comparison, the VTTC is defined for both approaches in the same way. However, and this is one of the key points of the present work, the estimates from both models may differ: any difference would be an empirical matter, related to how the error terms are conceived in each model.

## 5.2 Model specification 2: Logarithmic base models (multiplicative error terms)

The second specification considers also a base model, but now with multiplicative error terms. Introducing error terms in an additive way is not a requirement of microeconomic theory (Harris and Tanner, 1974). The intuition beyond suggesting multiplicative errors over additive errors is the following: relative differences between the utilities of the choice options may be more important for decisions than absolute differences (see Fosgerau and Bierlaire, 2009). In order to estimate models with multiplicative error terms, Fosgerau and Bierlaire (2009) suggest a logarithmic transformation of the utility function. This allows the use of common software. The counterpart logarithmic base specification can be derived for both approaches as follows:

### RU approach

$$\begin{cases} U_1 = \beta_c \left( \frac{\beta_t}{\beta_c} * t_1 + c_1 \right) * \varepsilon_1 \\ U_2 = \beta_c \left( \frac{\beta_t}{\beta_c} * t_2 + c_2 \right) * \varepsilon_2 \end{cases} \quad (22)$$

$$\begin{cases} U_1' = \mu * \ln(\text{VTTC} * t_1 + c_1) + \varepsilon_1' \\ U_2' = \mu * \ln(\text{VTTC} * t_2 + c_2) + \varepsilon_2' \end{cases} \quad (23)$$

### RV approach

$$\begin{cases} U_1 = \mu * \text{BVTTC} * \varepsilon_1 \\ U_2 = \mu * \text{VTTC} * \varepsilon_2 \end{cases} \quad (24)$$

$$\begin{cases} U_1' = \mu * \ln(\text{BVTTC}) + \varepsilon_1' \\ U_2' = \mu * \ln(\text{VTTC}) + \varepsilon_2' \end{cases} \quad (25)$$

Where:

$\beta_c$  is normalized to 1 for identification reasons in the RU approach.

$$U_i' = \ln(U_i)$$

$$\varepsilon_i' = \mu \ln(\varepsilon_i)$$

$\mu$  is a scale parameter associated with  $\varepsilon_i$

With:

$$VTTC = \frac{\beta_t}{\beta_c} = \beta_0 \quad (26)$$

Note, however, that in the multiplicative RV approach equation (26) implies that the error term is not interpreted as part of the individuals' preferences. Given that in the RV approach the error relates to the VTTC, one could assume that the calculation of the mean VTTC should take the error into account (e.g. Fosgerau et al., 2007). In that case, if the error is part of individuals' preferences, then the VTTC should be calculated taking the logistic distribution of the error (the difference of two type-I GEV distributed error terms follows a logistic distribution) into account as follows:

$$VTTC = \exp[\ln(\beta_0) + \frac{1}{\mu}(\varepsilon_1' - \varepsilon_2')] \quad (27)$$

This expression can be calculated using simulation for the logistic distributions. Additionally, given that those distributions will be unbounded, it is necessary to make an assumption for the VTTC values which our data (given mainly by the range of BVTTC) does not support (see Börjesson et al., 2012). One possibility is to censor the VTTC distribution, restricting it to be close to the BVTTC range.

### 5.3 Model specification 3: Observed heterogeneity (covariates)

The third specification builds on the base logarithmic specification above (both approaches in logarithms provided better model fit than when constructed linearly). Now, the VTTC may vary with individuals' and trip characteristics. Models can be extended to accommodate more precise definitions of the VTTC based on observed heterogeneity. Income and individuals' reported levels of current travel cost and current travel time are selected for this extension. The VTTC that enters equations (23) and (25) is now defined as:

$$VTTC = e^{\beta_0 + \beta_{BC} \ln(\frac{C}{C_0}) + \beta_{BT} \ln(\frac{T}{T_0}) + \beta_I \ln(\frac{I}{I_0})} = \beta_0 * \left(\frac{C}{C_0}\right)^{\beta_{BC}} \left(\frac{T}{T_0}\right)^{\beta_{BT}} \left(\frac{I}{I_0}\right)^{\beta_I} \quad (28)$$

Where:

C = Current travel cost

C<sub>0</sub> = Reference level of current travel cost (e.g. average)

T = Current travel time

T<sub>0</sub> = Reference level of current travel time (e.g. average)

I = Income of the individual

$I_0$  = Reference level of income (e.g. average)

$$\beta_0 = \frac{\beta_t}{\beta_c}.$$

The VTTC has been defined using two identical expressions in equation (28). The inclusion of each covariate divided by a reference value allows the researcher to readily obtain a VTTC at the reference levels of the covariates (e.g. sample average). The coefficients on the covariates can be directly interpreted as elasticities. The essence of this particular way of defining the VTTC was employed in both the UK and Danish studies. However, defining the VTTC using the exponential function is more beneficial for estimation because it ensures positivity of the VTTC (especially important when logarithms are employed). For the reference values, an approximation to the sample average value has been used for all covariates. For the UK dataset the reference values are: ( $C_o = 440$ ,  $T_o = 60$ ,  $I_o = 27$ ). For the Danish dataset: ( $C_o = 5360$ ,  $T_o = 45$ ,  $I_o = 26$ ). The key comparisons of our work are carried out within-country: therefore it is safe to simply work at sample averages in both datasets. Of course, many other exogenous individual and trip characteristics could be used as explanatory variables for the VTTC (e.g. gender, age class, occupation, congestion, etc.). However, the target of this model specification is to add only a few critical covariates rather than conduct a full specification search. The three selected covariates typically account for a great amount of observed variation in VTTC studies.

Again, equation (27) would need to be applied if the errors are assumed to be part of the travellers' preferences. For model specifications 2 and 3, three estimates of the VTTC, depending on the interpretation on the logistic error and the censoring assumption, will be shown.

#### **5.4 Model specification 4: Random heterogeneity**

The last model specification considered in this work extends the previous one to account for unobserved random heterogeneity. It is common to find additional variability in the VTTC that the models have not yet accounted for through covariates. This can be introduced by adding a random parameter which follows a particular distribution to the VTTC definition. Let us assume the VTTC follows a log-normal distribution across travelers:

$$VTTC = e^{\beta_0 + \beta_{BC} \ln\left(\frac{C}{C_0}\right) + \beta_{BT} \ln\left(\frac{T}{T_0}\right) + \beta_I \ln\left(\frac{I}{I_0}\right) + u} \quad (29)$$

Where  $u$  is a random parameter that follows a normal distribution  $N(0, \sigma)$  and hence the VTTC is log-normally distributed across individuals, with mean:

$$E(VTTC) = e^{(\beta_0 + \beta'X)} e^{\left(\frac{\sigma^2}{2}\right)} \quad (30)$$

Where  $\sigma$  is the standard deviation of  $u$  and  $X$  represents the set of covariates. Given the definition in (28), at the reference values chosen for the covariates, the mean is simply calculated as:

$$E(\text{VTTC}) = e^{\beta_0} e^{\left(\frac{\sigma^2}{2}\right)} \quad (31)$$

Other distributions could also be tested, but we have chosen the log-normal distribution as it has been extensively used in this field (e.g. Fosgerau et al., 2007, Borjesson and Eliasson, 2014). Our aim in this section is not to identify the best distribution for the VTTC but to compare both models while accounting for random heterogeneity in some way.

### 5.5 Results

In this section the model estimation results are presented. All models have been estimated using Biogeme (Bierlaire, 2003). Tables 3 and 4 below show the results on the UK and Danish dataset respectively. For each dataset, the eight models are presented by pairs. The two approaches (RU and RV) are compared at four levels of model specifications. At the same time, the changes within each approach as the model specification improves are observed. For a more robust analysis, the standard errors of the estimated VTTC are also provided (see e.g. De Grange et al., 2015; Hole, 2007).

All estimated coefficients are significant at the 99% level of confidence. Surprisingly, the overall results of interest are very similar in both datasets. In all cases, models from the RV approach fit the data better than their counterparts based on the RU approach: although the models are not nested, the final Log-Likelihood improves significantly with the same number of parameters. Therefore, the empirical issue of selecting the modelling approach favours the RV approach, in line with existing literature (Börjesson and Eliasson, 2014; Hultkranz et al., 1996; Fosgerau, 2007). This means that, given a set of travellers' choices on time-cost tradeoffs, it is better to incorporate the error term assuming that the difference between VTTC and BVTTC (rather than the utility difference between travel options) is distributed with constant variance.

Within each approach (RU and RV), the use of logarithms improves the model fit. Since they are justified as a mean to introduce multiplicative error terms, this finding suggests that relative differences between utilities ( $V_i$ ) are more important than absolute differences for individuals' choices. (Fosgerau and Bierlaire (2009) report similar findings). However, this was only tested with a base model. We are aware that other works (e.g. Significance et al., 2013) have found that logarithms may not improve linear specifications when the utility specification is refined (i.e. accounting for significant sources of heterogeneity)<sup>1</sup>. On top of this, as usual, the major improvement in model fit comes from the introduction of the random parameter  $u$ .

---

<sup>1</sup> Testing the comparison between linear and logarithmic models under more refined model specifications would be an interesting extension of the empirical work presented here.





Table 3. Results - UK dataset (Standard errors for the VTTC in brackets)

	1. Linear				2. Logarithms			
	RU		RV		RU		RV	
	Est.	t-test	Est.	t-test	Est.	t-test	Est.	t-test
$\beta_c$	1	na	na	na	1	na	na	na
$\beta_0$	4.89	18.64	3.22	11.76	3.71	22.38	2.75	23.71
$\mu$	-0.0138	-20.81	0.115	24.03	-6.42	-23.12	0.79	33.15
<b>VTTC pence/min</b>	4.89 (0.26)		3.22 (0.27)		3.71 (0.17)		2.75 (0.12)	
							4.28*	5.1**
<b>Obs.</b>	10598		10598		10598		10598	
<b>Parameters</b>	2		2		2		2	
<b>Null LL</b>	-7345.974		-7345.974		-7345.974		-7345.974	
<b>Final LL</b>	-6746.152		-6570.224		-6690.042		-6465.961	
<b>Adj. Rho<sup>2</sup></b>	0.081		0.105		0.089		0.120	

	3. Logarithms + Covariates				4. Logs + Covariates + Random Heterogeneity			
	RU		RV		RU		RV	
	Est.	t-test	Est.	t-test	Est.	t-test	Est.	t-test
$\beta_c$	1	na	na	na	1	na	na	na
$\beta_0$	1.70	30.23	1.30	28.25	1.58	28.64	1.29	28.11
$\mu$	7.39	25.26	0.859	34.24	11.5	24.06	1.09	33.00
$\beta_{BC}$	0.470	8.78	0.431	7.57	0.431	7.45	0.428	25.29
$\beta_{BT}$	-0.362	-4.81	-0.196	-2.68	-0.279	-3.50	-0.189	-2.61
$\beta_I$	0.273	5.13	0.411	8.06	0.344	6.40	0.382	7.77
$\sigma$	na	na	na	na	1.07	21.22	1.11	25.29
<b>VTTC pence/min</b>	5.47 (0.31)		3.67 (0.17)		8.61 (0.57)		6.72 (0.42)	
<b>Obs.</b>	10598		10598		10598		10598	
<b>Parameters</b>	5		5		6		6	
<b>Null LL</b>	-7345.974		-7345.974		-7345.974		-7345.974	
<b>Final LL</b>	-6607.502		-6300.028		-6306.561		-5910.137	
<b>Adj. Rho<sup>2</sup></b>	0.100		0.142		0.141		0.195	

\* Logistic error is part of preferences (VTTC distribution censored at 25p/min.).

\*\* Logistic error is part of preferences (VTTC distribution censored at 35p/min.).

Table 4. Results - Danish dataset (Standard errors for the VTTC in brackets)

	1. Linear				2. Logarithms			
	RU		RV		RU		RV	
	Est.	t-test	Est.	t-test	Est.	t-test	Est.	t-test
$\beta_c$	1	na	na	na	1	na	na	na
$\beta_0$	37.9	14.6	20.5	11.9	31	13.3	18.7	23.91
$\mu$	-0.058	-14.5	0.0169	28.7	-3.14	-18.47	0.711	35.36
<b>VTTC DKK/hour</b>	37.95 (2.59)		20.5 (1.72)		18.6 (1.4)		18.7 (0.78)	
							22.2*	28.84**
<b>Obs.</b>	17020		17020		17020		17020	
<b>Parameters</b>	2		2		2		2	
<b>Null LL</b>	-11797.4		-11797.4		-11797.4		-11797.4	
<b>Final LL</b>	-11378.7		-10807.6		-10922.3		-10763.2	
<b>Adj. Rho<sup>2</sup></b>	0.035		0.084		0.074		0.087	

	3. Logarithms + Covariates				4. Logs + Covariates + Random Heterogeneity			
	RU		RV		RU		RV	
	Est.	t-test	Est.	t-test	Est.	t-test	Est.	t-test
$\beta_c$	1	na	na	na	1	na	na	na
$\beta_0$	4.33	62.25	3.89	75.79	4.12	68.27	3.89	73.74
$\mu$	4.41	20.37	0.768	36.3	10.3	24.4	1.06	34.84
$\beta_{BC}$	0.571	6.51	0.701	9.44	0.581	7.00	0.705	9.23
$\beta_{BT}$	-0.48	-3.87	-0.643	-6.06	-0.451	-3.67	-0.633	-5.77
$\beta_I$	0.501	6.63	0.638	9.76	0.611	8.47	0.633	9.74
$\sigma$	na	na	na	na	1.49	26.8	1.47	30.21
<b>VTTC DKK/hour</b>	45.57 (3.15)		29.35 (1.31)		112.08 (9.11)		86.45 (7.13)	
			26.65*	35.8**				
<b>Obs.</b>	17020		17020		17020		17020	
<b>Parameters</b>	5		5		6		6	
<b>Null LL</b>	-11797.4		-11797.4		-11797.4		-11797.4	
<b>Final LL</b>	-10748.8		-10313.8		-9690.48		-9185.81	
<b>Adj. Rho<sup>2</sup></b>	0.088		0.125		0.178		0.221	

\* Logistic error is part of preferences (VTTC distribution censored at 200DKK/h).

\*\* Logistic error is part of preferences (VTTC distribution censored at 300DKK/h).

The following graphs in figures 1 and 2 summarise the mean VTTC across the eight model specifications (for the RV approach where logarithms are used with type-I GEV errors, the VTTC selected for the graph is that where errors are assumed to be part of preferences and the simulated VTTC distribution was censored to the range of BVTTC in the data):

Figure 1. VTTC results - UK dataset

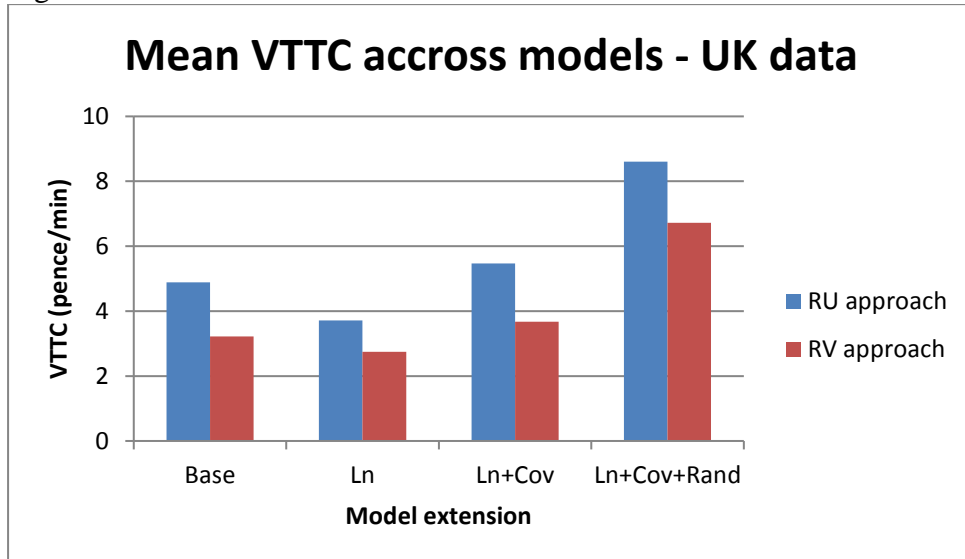
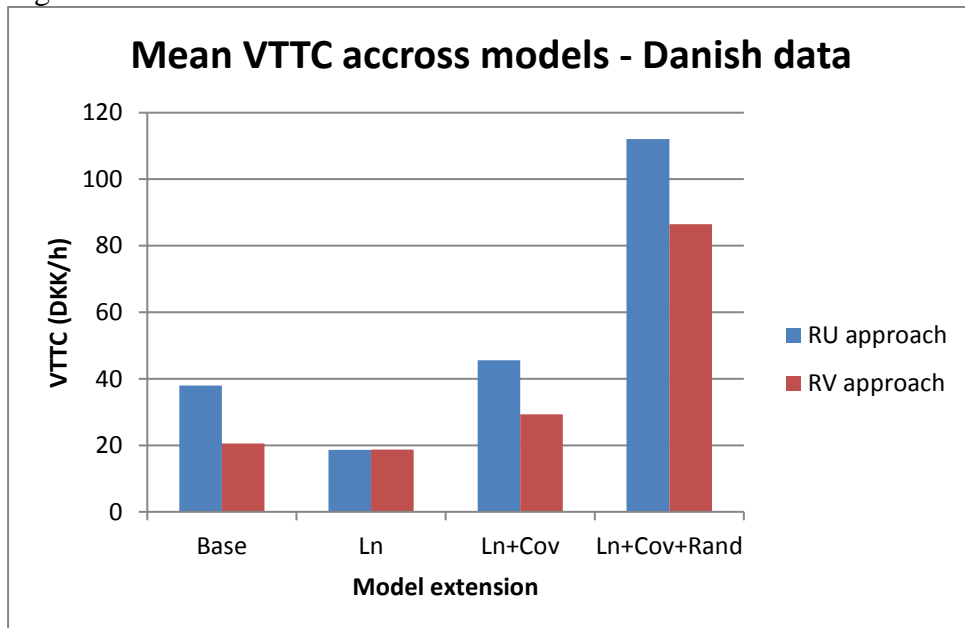


Figure 2. VTTC results - Danish dataset



In both countries, the RV approach gives systematically lower VTTC estimates at all levels of model sophistication (with the exception of the base logarithmic specification in the Danish dataset and also in the UK dataset when the errors are assumed to be part of the preferences for the selected levels of censoring). The use of logarithms decreases the VTTC estimates compared to the linear base specification in the RU approach. This would also be true for the RV approach unless the errors are taken into account as part of the preferences, which is probably the correct assumption. Consistently with other works in the field, the

introduction of observed and, especially, unobserved heterogeneity significantly increases the mean VTTC, as it allows the model to capture the right tail of the highly skewed VTTC distribution (Börjesson and Eliasson, 2014). The most recent similar VTTC study (Significance et al., 2013) found this only for unobserved heterogeneity.

Surprisingly, the variation of the VTTC across the eight model specifications is remarkably similar in both datasets (see figures 1 and 2), which were collected using the same basis for the SC design. The similarity exists regardless of the interpretation of the logistic error in two of the four RV models. Furthermore, the effects of all covariates occur in the same direction in both datasets (same sign of parameters) and are only slightly more accentuated in the Danish dataset. This leaves us with a feeling that the SC designs might be playing a relevant role in the results.

## 5.6 Recommendations

Acknowledging that the VTTC can be modelled in many different ways, the *RV approach* seems very promising and, where practical, should at least always be considered as an option for modelling. The nature of referendum data makes the *RV approach* a very reasonable option, which has been confirmed in this paper. Classical utility settings (i.e. *RU approach*) are likely to contain heteroskedastic error terms that need correction. In the case of more complex choice scenarios (e.g. more attributes or alternatives) where a valuation threshold (the BVTTC) cannot be observed and *RU approach* is employed, correcting for heteroskedasticity is highly recommended. The researcher should look for potential causes of heteroskedasticity and adjust the models accordingly (see e.g. Daly and Carrasco, 2009; and Munizaga et al., 2000). In the time-cost trade off case analysed here, a correction term for heteroskedasticity would divide the utility function of the *RU approach* by the change in travel time ( $\Delta t$ ). The biases in the VTTC can be significant if the right form of heteroskedasticity is not identified.

Additionally, although logarithms seem to fit the data better, testing both linear and logarithmic specifications seems a sensible approach. Although it has not been implemented in this work, it is also possible to test intermediate options between linear and logarithmic transformations, such as a Box-Cox transformation (see Daly and Tsang, 2009).

The differences in the VTTC between *RU* and *RV* are of a factor of 1.3 to 1.85. The magnitude of these differences can potentially have severe implications on the evaluation of transport projects. For example, the economic case for the High-Speed Rail (HS2) in the UK shows that the Cost-Benefit Ratio (CBR) of a project can be very sensitive to differences in the VTTC of the magnitude reported here (Department for Transport, 2013).

In relation to the use of the *RV* model for appraisal, in essence, what is needed from the (behavioural) model is an estimate of the VTTC. Both *RU* and *RV* are able to provide this and there is nothing material to distinguish the two approaches. Given the equivalence of models in theory, both emerge from microeconomic theory and hence both could safely be used to derive behavioural values that can then be transformed into appraisal values (see Mackie et al., 2003).

Although the results of our work would point towards the recommendation of the *RV approach*, we believe that more research is needed in order to fully understand what causes the differences in results between *RU* and *RV*. Especially, it should be borne in mind that the only difference between the two approaches lies in how the error terms are related to the

observable part of the model. The use of simulated data could be a very useful tool to shed more light on this debate.

Several questions are left open. Why is the VTTC generally lower with the RV approach? If the RV approach actually explains choices better, has the VTTC been biased (overestimated) in applications using RU approach that did not correct for heteroskedasticity? And why do individuals' preferences seem so similar in two different countries? What can be said about travellers' behaviour in light of the evidence provided by RU and RV approaches? How would our results change if different data collection methods were employed? Further research regarding SC designs and methods for VTTC estimation is encouraged.

## 6 Conclusions

In this paper, two popular approaches for the estimation of the VTTC have been identified, related and compared. The focus is placed on official national VTTC studies using data from travellers' choices in binary time/money trade-offs. The theoretical relationship between the two approaches, namely Random Utility (RU) approach and Random Valuation (RV) approach, has been shown. They simply differ in the assumptions regarding the introduction of the error term, and so neither is theoretically preferred to the other. An extensive empirical comparison using two datasets from the national studies in the UK and Denmark has revealed significant differences in the VTTC estimates provided by the RU and RV models respectively. The analysis has also led us to conclude that the RV approach should be preferred, regardless the level of model sophistication employed, since it always provides a much better fit to the data. This paper is the first to show a direct empirical comparison of the two approaches. Several levels of model sophistication have been considered, in order to disentangle the impact of certain factors such as the use of logarithms and the introduction of observed and random heterogeneity. The VTTC is, in general, systematically lower using the RV approach, which highlights the risk of significant biases if the correct form of error heteroscedasticity is not employed. The magnitude of the differences, of a factor of 1.3 to 1.8, can have important implications in the evaluation of transport schemes. Finally, a surprisingly similar pattern of results across models in both datasets, based on a similar SC design, is found. Several questions are left open. Further research on the current techniques to collect data and estimate the VTTC would be welcome. In particular, simulated data could be very useful to shed light on this topic.

## Acknowledgments

This work was supported by the Economic and Social Research Council (ESRC) through the ESRC White Rose Doctoral Training Centre. We would also like to thank Professor Maria Borjesson, Professor Lars Hultkrantz, Professor Gerard de Jong, Professor Caspar Chorus and Andrew Daly for helpful comments during this work. The authors remain responsible for errors and interpretations contained in the paper.

## References

**Accent Marketing and Research and Hague Consulting Group (1996).** The Value of Time on UK Roads, *The Hague*.

**Becker, G.S. (1965).** A Theory of the Allocation of Time. *The Economic Journal*, 75 (299), pp.493-517.

**Bierlaire, M. (2003).** BIOGEME: A free package for the estimation of discrete choice models *Proceedings of the 3rd Swiss Transportation Research Conference*, Ascona, Switzerland.

**Block H.D. and Marschak, J. (1974).** Random orderings and stochastic theories of responses. In *J. Marschak, Economic Information, Decision and Prediction: Selected Essays*, Vol.1, 1960, pp. 172-217. Dordrecht: D. Reidel

**Börjesson, M., Fosgerau, M. and Algiers (2012).** Catching the Tail: Empirical identification of the Distribution of the Value of Travel Time. *Transportation Research A*, Vol. 46, No. 2, 2012, p. 378-391

**Börjesson, M. and Eliasson, J. (2014).** Experiences from the Swedish Value of Time study. *Transportation Research Part A*, 59 pp.144-158.

**Burge, P., and Rohr, C. (2004).** DATIV: SP Design: Proposed approach for Pilot Survey. *Tetra-Plan in Cooperation with RAND Europe and Gallup A/S*.

**Cameron, T.A. (1988).** A new paradigm for valuing non-market goods using referendum data: maximum likelihood estimation by censored logistic regression. *Journal of Environmental Economics and Management*, 15, pp.355-79.

**Cameron, T.A. and James, M.D. (1987).** Efficient estimation methods for 'closed-ended' contingent valuation surveys. *The Review of Economic and Statistics*, Vol.69, No.2, pp.269-276

**Daly, A. and Carrasco, J. (2010).** The influence of trip length on marginal time and money values, in *The Expanding Sphere of Travel Behaviour Research: Selected papers from the proceedings of the 11th Conference on Travel Behaviour Research*, Kitamura, R. et al. (eds.), Emerald Books.

**Daly, A., Tsang, F. and Rohr, C. (2014).** The Value of Small Time Savings for Non-business Travel. *Journal of Transport Economics and Policy*

**Daly, A., and Zachary, S. (1975).** Commuters' values of time. *Report T55. Local Government Operational Research Unit, Reading*.

**Daly, A. and Tsang, F. (2009).** Improving understanding of choice experiments to estimate values of travel time. *Association for European Transport*.

**Department for Transport (2013).** Economic Case for HS2. Accessed online 13/08/2015: [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/365065/S\\_A\\_1\\_Economic\\_case\\_0.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/365065/S_A_1_Economic_case_0.pdf)

**De Grange, L.; Fariña, P. and Ortúzar, J. (2015).** Dealing with collinearity in travel time valuation, *Transportmetrica A: Transport Science*, 11 (4), 317-332.

**DeSerpa, A.C. (1971).** A Theory of the Economics of Time. *The Economic Journal*, Vol.81, No.324, pp.828-846.

**Fosgerau, M. (2007).** Using Non-parametrics to specify a model to measure the value of travel time. *Transportation Research A*, Vol. 41(9), pp. 842-856.

- Fosgerau, M., Hjorth, K. and Lyk-Jensen, S. (2007b).** The Danish Value of Time Study: Results for Experiment 1.
- Ramjerdi, F., Flügel, S., Samstad, H., and Killi, M. (2010).** Value of time, safety and environment in passenger transport – Time. *Transportøkonomisk institutt*
- Fosgerau, M., Hjorth, K. and Vincent, S. (2007).** An approach to the estimation of the distribution of marginal valuations from discrete choice data. *Munich Personal RePEc Archive, Paper No.3907*
- Fosgerau, M. and Bierlaire, M. (2009).** Discrete Choice Models with Multiplicative Error Terms. *Transportation Research B*.
- Hague Consulting Group (1998).** Value of Dutch Travel Time Savings in 1997 - Final report, a report for Rijkswaterstaat - AVV. *Hague Consulting Group, The Hague*.
- Harris, A. J. and Tanner, J. C. (1974).** Transport Demand Models Based on Personal Characteristics, *TRRL Supplementary Report 65UL*.
- Hensher, D.A. and Greene, W.H. (2011).** Valuation of travel time savings in WTP space in the presence of taste and scale heterogeneity. *Journal of Transport Economics and Policy*, 45 (3), 505–525.
- Hole, A.R. (2007).** A comparison of approaches to estimating confidence intervals for willingness to pay measures. *Health Economics*, 16 (8), 827-840.
- Hultkranz, L., Li, C., Lindberg, G. (1996).** Some problems in the consumer preference approach to multimodal transport planning. *CTS Working Paper*.
- Jara-Díaz, S.R. (2002).** The Goods/Activities Framework for Discrete Travel Choices: Indirect Utility and Value of Time. In *Mahmassani, H.S. (editor): Perpetual Motion: travel Behavior Research Opportunities and Application Challenges. Elsevier Science*
- Jara-Diaz, S.R. (2003).** On the Goods-Activities Technical Relations in the Time Allocation Theory. *Transportation 30 (3), 2003, pp.245-260*
- Mackie, P., Wardman, M., Fowkes, A.S., Whelan, G., Nellthorp, J. & Bates J.J. (2003).** Values of Travel Time Savings in the UK. Report to Department for Transport. *Institute for Transport Studies, University of Leeds & John Bates Services, Leeds and Abingdon*.
- McFadden, D. (1974).** Conditional logit analysis of quantitative choice behavior. *Frontiers in econometrics, ed by P.Zarembka, Academic Press, New York, p.105-142*.
- Munizaga, M., Heydecker, B. and Ortuzar, J. (2000).** Representation of heteroskedasticity in discrete choice models. *Transportation Research Part B 34, pp.219-240*
- Significance, VU and Bates (2013).** Values of time and reliability in passenger and freight transport in the Netherlands. *Report for the Ministry of Infrastructure and the Environment*.
- Small, K. (2012).** Valuation of Travel Time. *Economics of Transportation 1, p.2-14*.



**Train, K. and McFadden, D. (1978).** The Goods/Leisure Tradeoff and Disaggregate Work Trip Mode Choice Models. *Transportation Research*, Vol.12, pp.349-353

**Train, K. (2009).** Discrete Choice Methods with Simulation. (2 ed.). Cambridge, UK: Cambridge University Press

**Wardman, M. (1998).** The Value of Travel Time: a Review of British Evidence. *Journal of Transport Economics and Policy*, Vol.32, 1998, pp.285-316