FUNCTIONAL APPROXIMATIONS TO ALTERNATIVE-SPECIFIC CONSTANTS IN TIME-PERIOD CHOICE-MODELLING

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ABSTRACT

This paper addresses a specific problem that arises in the application of time period choice models to long term forecasting. The need to specify such models in a form that is independent of individual scheduling information leads to a proliferation of highly interdependent constants in the model specification with serious consequences for identification and interpretation of model parameters. We propose an alternative approach which involves the parameterisation of this set of constants and test this approach using a number of alternative parameterisation approaches. The data used in these tests come from a recent stated preference exercise carried out in the Netherlands. The results indicate that this approach can yield significant improvements in applicability, with minimal degradation in model performance. The approach has applications in a host of other areas, including the analysis of route choice and spatial choice process.

INTRODUCTION & BACKGROUND

Alternative Specific Constants (ASCs) play an important role in the use of random utility models (see for example Train, 2003). Indeed, they capture the mean of the error terms of the utility functions. As such, they are critical in order to properly reproduce market shares when the models are used for forecasting. The estimated values of ASCs are affected by sampling bias (except when using exogenous sampling strategies with Generalized Extreme Value models), and by data collection methods such as stated preference (SP) surveys. Also, when the number of alternatives is high, the model may be saturated by ASCs, suffer from overfitting and, consequently, lose its forecasting power. Another source for biased ASCs is the estimation of models based on sampling of alternatives.

One area of research in which the number of constants can be very high, with high levels of correlation between related constants, is the modelling of time period choice. This has become increasingly important over recent years. Indeed, since both congestion and many of the consequent policy responses to it have strong time of day related components, as the salience of congestion as a policy concern has grown in recent years, so has the importance of developing theoretically sound yet practically applicable approaches to modelling the timing of travel, particularly for departure times of trips during congested peak periods.

A number of different modelling approaches have been proposed in the literature. Building on ideas originally proposed by Vickrey (1969), several authors have presented frameworks in which the choice of departure time is modelled deterministically and as a continuous quantity (e.g., Arnott *et al*., 1990; de Palma *et al*., 1997; Hyman, 1997; van Vuren *et al*., 1999). A related set of studies have modelled departure time jointly with route choice using model systems in which a continuously variable choice of departure time is linked to a discrete choice of route (e.g., Mannering *et al*., 1990; Mahmassani and Chang; 1985, Mahmassani *et al*., 1991). More recently, interest has also developed in formulating continuous models of departure time choice within the framework of hazard based duration models (e.g., Bhat and Steed, 2002; Wang, 1996). In addition, models of individual trip departure time are also increasingly being embedded in more general models of activity choice and scheduling behaviour (see, e.g., Ashiru *et al*., 2003 and the work reviewed therein).

However, the dominant approach to modelling departure time, both in the academic literature and in practice, involves re-formulating the underlying continuous departure time choice problem as a choice problem involving a finite number of discrete time periods and modelling the choice between these periods within the framework of discrete choice analysis. This approach, first proposed in the work of Cosslett (1977) and Small (1982), has been widely applied using both revealed preference data (e.g., Abkowitz, 1981; Athanassiou and Polak, 2001; Bhat 1998; Bradley *et al*., 1998; Chin, 1990; Hendrickson and Plank, 1984; McCafferty and Hall, 1982; Small, 1982,1987) and stated preference data (e.g., Bates *et al*., 1990; de Jong *et al*., 2003; Polak *et al*., 1991; Polak and Jones, 1994).

Models based on this *time period choice* approach are typically formulated in terms of a trade-off between, on the one hand, the time of day varying travel times and costs experienced by travellers and on the other hand, travellers' inherent preferences for undertaking certain activities at certain times of day. The characterisation of these temporal preferences is challenging and the most commonly used approach is to use a version of the concept of schedule delay (Vickrey, 1969) to quantify the loss in utility associated with shifting a departure earlier or later relative to the preferred (or, more often, the actual) time of departure of the existing trip. This approach works well in diagnostic and exploratory modelling but when time period choice models are being used in long term forecasting applications, the embedding of this sample type information in the utility function can be highly problematic, since the notion of 'existing trip' is not meaningful in a forecasting context and, moreover, the characteristics of forecast trips are typically not available at the same level of temporal precision as that available in the estimation sample dataset. Hence, when time period choice models are developed for forecasting applications, temporal preferences of travellers are captured by the use of a set of constants, associated with the different time-periods. Depending on the degree of granularity employed in the definition of the time periods, this approach can lead to significant problems with identification and interpretation, as well as heightened computational cost. These problems are compounded when time period choice models are specified in terms of tours rather than trips (e.g., de Jong *et al*., 2003; Polak and Jones, 1994) since the number of possible time period constants increases substantially.

Similar issues apply in many other areas of research where a large number of constants are to be identified, such as in the analysis of spatial choice or route choice processes. In this paper, we propose to address these problems by using an explicit modelling of the ASCs, based on the idea of a functional approximation to the distribution of the set of constants across alternatives. Such functional approaches not only reduce the number of parameters to be identified, but also avoid some of the issues of identification, and facilitate interpretation of the results. Furthermore, they potentially reduce the influence of SP errors on the estimation of ASCs when using data collected through SP surveys.

The remainder of this paper is organised as follows. In the next section, we briefly describe the data, before moving on to model specification in section 3. Section 4 presents the results of the modelling analysis, with model validation and conclusions being the topics of section 5 and 6 respectively.

DATA

This paper makes use of data collected in 2000 for the development of the Dutch National Model System (c.f. de Jong et al., 2003). An initial Revealed Preference (RP) survey was conducted to select a sample of respondents for the follow-up SP survey. The RP survey included rail-travellers as well as car-travellers, contacted at a selection of sites across The Netherlands, concentrating on areas where road and rail congestion was encountered in peak-period journeys. In the ensuing SP survey, respondents were presented with four alternatives in each choice-situation, three alternatives using their current mode of travel, and one alternative involving a mode-change. The choice thus involves the re-timing of an existing tour or the switch of this tour to an alternative mode. The three alternatives on the current mode are described as “retimed earlier”, “base”, and “retimed later”, with the “base” alternative being close in departure time to the actual observed tour. Except for non-home-based business travellers, the travel alternatives were presented to respondents in the form of complete tours, comprising explicit and linked outbound and return legs. Respondents were presented with 14 SP replications, spread over two games. The travel purposes distinguished were commuting, business, education, and other. This dataset has recently been used in the estimation of Error-Components Logit models by Hess et al. (2004). In the present analysis, we concentrate on car-commuters, using a division into commuters with flexible working hours, and commuters with inflexible working-hours.

MODEL SPECIFICATION

In models of time period choice destined for the use in forecasting systems, the aggregate scheduling information is generally captured through the use of a set of constants, associated with the different time-periods. The *usual* specification uses a separate constant for each possible combination of outbound and return time period, hence indirectly also capturing effects associated with activity duration. However, with *N* separate time periods, this leads to a requirement for *N(N+1)/2* constants, of which *N(N+1)/2-1* can be identified. With the typically used 24 1-hour time periods, this thus leads to a total of 299 constants that need to be estimated just to represent the choice of time period. This number increases dramatically with *finer* specifications of the time periods, compounding problems of computational cost and parameter identification. An alternative approach consists of the use of two separate sets of constants for outbound and return time periods. The independence between the two sets however leads to the requirement of an additional set of constants representing activity duration. In the 1-hour example, this thus leads to three sets of 24 constants, where 23 constants are estimated in each set. Despite the fact that this leads to a reduction in the (maximum) number of estimated constants from 299 to 69, the approach is still computationally expensive and problems of identification can remain, especially in the presence of time periods never chosen, or time periods always chosen when available.

In this paper, we discuss an alternative approach, based on the idea of a functional form for the constants. These functional approaches will inevitably lead to poorer model fit than an approach using a full set of constants, but fewer problems with identification and significance levels should occur.

In a properly specified time period choice-model, the constants associated with the different time periods will reflect the distribution of travellers across time periods. With an appropriate size for the time periods, it can be expected that these distributions follow some form of systematic pattern. As an example, for commuting, it can be expected that there is a peak in the distribution of the outbound constants sometime between 7am and 9am, with a corresponding peak in the distribution of the return constants between 5pm and 7pm. The differences in the shape of the distribution to the left and right of these *base* periods can give important insights into travel and more specifically rescheduling behaviour, and can be seen as a reflection of the flexibility of the working-hour arrangements.

With any approach using constants to capture time period choice, it is crucial to reduce the impact of artefacts of the SP survey on these constants. In the present analysis, three additional ASCs were hence defined on the basis of the SP design; associated with the *retimed-early*, *retimed-late* and *mode-shift* alternatives; such that the constant for the *base* alternative is normalised to zero. Finally, in the presence of quota-based data, and in the absence of a reliable re-weighting strategy, it is crucial to acknowledge that the results produced in this analysis relate to the sample used and should not be generalised; this does however not translate to the theoretical findings and recommendations. Similarly, the results are affected by the fact that not all time periods are available to all respondents.

In this paper, we make use of two groups of functional forms for the distribution of the constants. In the first group, we make an a priori assumption that there is a single peak in the distribution, with (possibly asymmetrical) slopes to either side, while, in the second group, we allow for the possibility of multiple peaks. Both sets of approaches ignore the potential interaction between the distributions along the three dimensions; the analysis of such interactions is an important avenue for further research.

With time-period choice captured exclusively with the help of the constants, it is possible to reduce the number of alternatives from the number of time-periods (which in this case, in conjunction with the SP design, would have led to a total of 1,200 alternatives) to the number of SP alternatives (in this case four), with the help of an appropriate functional form, or the use of dummy variables. While this approach eases computational costs, it prevents a GEV-style treatment of correlation in the errors between alternatives in adjacent time periods. A preliminary analysis however revealed that such correlation levels are relatively low in the present dataset, which could be a reflection of the specific SP design. The theoretical findings of the paper clearly also apply to the case where a full set of time-period alternatives is used.

The free estimation package Biogeme (Bierlaire, 2003, roso.epfl.ch/biogeme) was used for the calibration of the models. This allows the specification of nonlinear utility functions in free form, as required in the present analysis.

Exponential formulation

The first of our proposed approaches makes use of an exponential transformation. Looking at the distribution of the constants referring to the outbound period, we define a base period B_{out}, and, for a given outbound period x, we set:

$$\delta_{o-} = max(B_{out}-x,0) \quad \text{and} \quad \delta_{o+} = max(x-B_{out},0).$$

The value of the function at time period x is then given by:

$$f(\delta_{o-},\delta_{o+}) = \exp(\beta_{o-}\cdot\delta_{o-}+\beta_{o+}\cdot\delta_{o+}),$$

where δ_{o-} and δ_{o+} are defined as above, and where the two shape parameters β_{o-} and β_{o+} characterise the slope to either side of the peak.

The formulation can be adapted for return time periods as well as for durations, where the duration was in our example calculated as the difference between the time period containing the return departure time and the time period containing the outbound arrival time. As such, the possible values along this dimension range from 0 to 23, instead of 1 to 24.

Aside from the issue of allowing only for a single peak, the main problem with this approach is that of finding an appropriate choice for the base period. While the parameters β_{o-} and β_{o+} are relatively straightforward to estimate, the estimation of the *base* parameters (e.g. B_{out}) is significantly more complicated. Indeed, these parameters interact in such a way with the arguments of the distribution (and its form) that significant problems with local optima are almost inevitably encountered in their estimation. As such, the base parameters generally need to be fixed *a priori*, with multiple runs used to determine the optimal location for the peak.

Power formulation

Even though the estimation of separate shape parameters β_{o-} and β_{o+} allows for different slopes to the left and the right of the peak, the use of the exponential transform imposes a strict a priori constraint on the behaviour of the functional form. An alternative approach uses a power function formulation, where the value of the function at time period x is given by:

$$f(\delta_{o-},\delta_{o+}) = \beta_{o-}\cdot(\delta_{o-})^{\lambda o-} + \beta_{o+}\cdot(\delta_{o+})^{\lambda o+},$$

where λ_{o-} and λ_{o+} need to be estimated in addition to β_{o-} and β_{o+}. Aside from being more flexible than the exponential formulation, the λ_{o-} and λ_{o+} have a specific interpretation, in that they are elasticities of $f(\delta_{o-},\delta_{o+})$ with respect to δ_{o-} (for $x<B_{out}$) and δ_{o+} (for $x>B_{out}$) respectively. Indeed, with $x<B_{out}$, the functional form reduces to:

$$f(\delta_{o-}) = \beta_{o-}\cdot(\delta_{o-})^{\lambda o-},$$

and we have:

$$\lambda_{o-} = [\partial f(\delta_{o-})/\partial\delta_{o-}]\cdot[\delta_{o-}/ f(\delta_{o-})].$$

A similar reasoning applies in the case where $x>B_{out}$. The estimation of the base parameters is hampered by similar problems to those faced in the case of the exponential formulation.

Empirical formulation

While the exponential and power formulations have advantages in terms of identification and parsimony over the approach using a full set of constants, they lack flexibility, by imposing a single-peak constraint. This formed the motivation behind the development of an alternative approach, where a middle-way between the parametric and the constants-only approach was chosen, based on the use of empirical distributions. Formally, let N give the total number of time periods, with Δ_s denoting the lowest represented time period, and Δ_e giving the highest represented time period, where these may differ from the first, respectively last time period. With K intermediary support points at values Δ_k ($k=1,\ldots,K$) defined such that

$$1\leq \Delta_s<\Delta_1<\ldots< \Delta_k <\ldots< \Delta_K<\Delta_e \leq N,$$

the value of the function at time period x, with $1<x<N$, is given by:

$$f(x)=\begin{cases}\beta_s+(\beta_1-\beta_s)\cdot\left(1-\dfrac{\Delta_1-x}{\Delta_1-\Delta_s}\right) & ,x\leq\Delta_1\\ \beta_{k-1}+(\beta_k-\beta_{k-1})\cdot\left(1-\dfrac{\Delta_k-x}{\Delta_k-\Delta_{k-1}}\right) & ,\Delta_{k-1}\leq x\leq\Delta_k,\, 2\leq k\leq K\\ \beta_K+(\beta_e-\beta_K)\cdot\left(1-\dfrac{\Delta_e-x}{\Delta_e-\Delta_K}\right) & ,x\geq\Delta_K\end{cases},$$

where β_k gives the value of the function at time period k, β_s gives the value of the function at the start-point and β_e gives the value of the function at the end-point. Intermediary values are found via interpolation, using linear segments between support points. With $K=N$-2, the formulation reduces to the approach using a full set of constants.
In this empirical formulation, the parameters Δ_k ($k=1,\ldots,K$) play an analogous role to the *base* period parameters in the exponential and power formulations discussed earlier and are subject to similar estimation difficulties such that it is of interest to keep these parameters fixed. Furthermore, problems can be caused by time periods that are always or never chosen, just as was the case with the approach using a full set of constants. The easiest way to estimate a model with this formulation seems to be to use an iterative approach starting with the biggest possible set of identifiable constants, and to drop constants in favour of interpolation if the results support such action. This approach can also be seen as a useful tool for testing the validity of a single peak assumption, and where this is supported, in the subsequent search of an appropriate *base* specification for the exponential and power formulations.

MODEL RESULTS

Basic MNL models

The first model fitted to the data was a simple Multinomial Logit (MNL) structure (c.f. Train, 2003), using no explicit treatment of the choice of time period, and fitted with the aim of finding an appropriate specification of utility and providing a base model against which to compare the more advanced models. Separate models were estimated for commuters with flexible and inflexible working hours. The main explanatory factors used were travel-time and travel-cost. Attempts were also made to include a frequency coefficient for the public transport alternative, but no significant effect could be identified, which can at least partly be explained by the use of only three levels of frequency in the survey. No gains in model fit could be made by the use of non-linear transforms. A number of interactions with socio-demographic attributes were explored, notably in the form of dummy variables reflecting the preference of certain population segments for the various SP alternatives. Attempts were also made to estimate separate rail-cost coefficients for commuters receiving full, partial, and no employer compensation for public transport travel. Finally, a continuous formulation was used where appropriate to account for the effect of income on certain marginal utilities. Specifically, letting y_i denote the household-income of individual i, with the average household-income in the population defined as $\bar{y}$, the utility-term associated with a given cost-variable x for this decision-maker is given by $\beta \cdot (y_i/\bar{y})^{\lambda} x$, where λ represents the elasticity of cost-sensitivity with respect to the relative income of individual i.
The results of the MNL analysis are summarised in Table 1. The fact that the population-level dummies for the retimed alternatives and the mode-shift alternatives are all negative reveals that, *ceteris paribus*, respondents prefer the base alternative, and hence, by implication, the observed (RP) alternative. The results show a slightly higher ratio between the constants for late and early departure in the case of commuters with inflexible working hours than for

commuters with flexible working hours, supporting results by Hess et al. (2004) that reveal far higher sensitivities to late arrival for commuters with non-flexible working hours than for commuters with flexible working hours. For travel cost, a significant coefficient for commuters with flexible working hours could only be identified for car-cost and uncompensated rail-travel (at the 71% level), while, for commuters with inflexible working hours, an additional coefficient was identified for semi-compensated rail-travel (at the 83% level, with an 80% level for car-cost). An income effect could only be identified for car-travel in the flexible commuters group, and for semi-compensated rail-travel in the inflexible commuters group, both times indicating decreasing sensitivity with increased income.

	FLEXIBLE		INFLEXIBLE	
Estimated parameters:	16		18	
Sample size:	2092		2101	
Final log-likelihood:	-1979.33		-1814.06	
Adjusted ρ^2:	0.2566		0.3215	
Constants	**Estimate**	**T-statistic**	**Estimate**	**T-statistic**
Earlier departure				
All travellers	-1.4592	-15.13	-1.8327	-16.82
Low education	-0.6469	-3.00	-	
Part-time	-0.5332	-2.64	-0.2765	-1.81
Working from home	-0.3223	-2.20	0.2885	1.47
Under sixty	-		0.3261	2.59
Later departure				
All travellers	-1.6991	-16.53	-2.3044	-13.74
Low education	-1.2649	-4.30	-0.3274	-1.65
Part-time	-0.3797	-1.86	-1.0000	-3.95
Working from home	0.4398	3.27	-	
Under sixty	-		0.4144	2.36
Mode-change				
All travellers	-2.0959	-4.82	-2.3587	-6.80
Low education	-		1.5449	6.43
Part-time	-		0.8387	3.28
Under sixty	0.3593	1.19	-0.6524	-3.14
Common rescheduling				
Under sixty	0.1529	1.59	-	
***Travel*-time(min)**				
Car	-0.0233	-9.40	-0.0191	-7.96
PT	-0.0308	-9.82	-0.0141	-5.77
Travel-cost (guilders)				
Car	-0.0161	-6.85	-0.0054	-1.27
λ_A	-0.3345	-8.13	-	
Semi-compensated PT	-		-0.0082	-1.36
λ_B	-		-2.0263	-4.33
Uncompensated PT	-0.0571	-1.05	-0.0242	-2.32
λ_C	-		-	

Table 1: MNL estimation results

MNL models with constants

The next step consisted of explicitly accounting for the time period choice dimension by using the approach based on three disjoint sets of 24 constants. To permit identification, a single constant was set to zero along each dimension, where the eight outbound constant (7am-8am) was selected for this, along with the seventeenth return constant (4pm-5pm), and duration constant number eight (8-9 hours). This is consistent with the average outbound journey time of around 1 hour, which would imply that commuters leaving in time period 8 arrive in time period 9, which together, with a return departure in time-period 17, would imply a duration between eight and nine hours. This is forthwith referred to as the 8-17-8 normalisation.
As expected, this approach led to identification problems, with a number of constants receiving very large positive (for alternatives always chosen when available) or negative values (for alternatives never chosen). Additionally, out of 69 estimated constants, 24 reduced to zero in the model for flexible commuters, with a corresponding figure of 16 in the model for commuters with inflexible working hours. These constants are associated with outbound periods, return periods, or durations that are not included in any of the SP choice-sets.

Outbound period	**Flexible**	**Inflexible**	***Return period***	**Flexible**	**Inflexible**	***Duration***	**Flexible**	**Inflexible**
1	-	-	1	-	-	0	0.58 (0.63)	-4.6 (-4.1)
2	-	-	2	-	-	1	0.65 (0.86)	-1.37 (-1.87)
3	-	-	3	-	-	2	0.52 (0.84)	-1.55 (-2.53)
4	-	-3.2 (-5.89)	4	-	-	3	-0.12 (-0.23)	-1.19 (-2.23)
5	-3.23 (-4.41)	-1.68 (-5.11)	5	-	-	4	-0.13 (-0.31)	-1.36 (-3.08)
6	-1.05 (-4.91)	-0.94 (-4.54)	6	-	-	5	-0.89 (-2.79)	-1.39 (-3.81)
7	-0.29 (-2.5)	-0.38 (-3.09)	7	-3.72 (-2.68)	-	6	-0.71 (-3.1)	-0.83 (-3.2)
8	**0**	**0**	8	-2.59 (-2.57)	2.33 (1.44)	7	-0.38 (-2.55)	-0.15 (-0.91)
9	-0.3 (-2.52)	-0.38 (-2.55)	9	-	-0.71 (-0.76)	8	**0**	**0**
10	-0.95 (-4.81)	-0.61 (-2.61)	10	-	-	9	-0.19 (-1.6)	-0.04 (-0.31)
11	-0.99 (-3.5)	-0.23 (-0.68)	11	-1.71 (-2.23)	0.03 (0.03)	10	-0.72 (-3.79)	-0.14 (-0.7)
12	-2.53 (-4)	0.81 (1.32)	12	-1.38 (-2)	0.55 (0.79)	11	-0.95 (-3.24)	-0.01 (-0.04)
13	-2.15 (-3.46)	1.49 (2.2)	13	-2.3 (-4.09)	-0.07 (-0.13)	12	-1.22 (-2.66)	-0.33 (-0.55)
14	-3.99 (-3.15)	1.34 (1.89)	14	-0.86 (-2.02)	-0.21 (-0.52)	13	0.24 (0.33)	-2 (-2.38)
15	-	-0.23 (-0.18)	15	-0.34 (-1.25)	0.22 (0.9)	14	-	-
16	-	-	16	0 (-0.01)	-0.21 (-1.42)	15	-	-
17	-	2.77 (1.52)	17	**0**	**0**	16	-	2.27 (1.81)
18	-	3.04 (1.92)	18	-0.02 (-0.12)	-0.28 (-1.93)	17	-	-
19	-	3.75 (2.32)	19	-0.21 (-1.03)	-0.98 (-3.76)	18	-	-
20	-	4.65 (2.94)	20	-0.81 (-2.62)	-1.22 (-3.47)	19	-	-
21	-	3.47 (2.19)	21	-1.35 (-2.88)	-1.52 (-3.14)	20	-	-
22	-	-	22	-1.24 (-1.82)	-2.03 (-3.31)	21	-	-
23	-	-	23	-	-2.1 (-2.85)	22	-	-
24	-	-	24	-	-2.15 (-2.65)	23	-	-

Table 2: Estimates of constants for fully-specified model with extra availability conditions

A set of availability conditions was then introduced to deal with the identification issues. For this, in the flexible model, alternatives associated with outbound constants 4, 15, 16, and 17, return constants 1, 6, 9, 10, and 23, and duration constant 14 were made unavailable. In the inflexible model, the unavailability constraint was applied to alternatives associated with outbound constants 1, 16 and 22, return constants 7 and 10, and duration constants 14, 15 and 17. The sample size for inflexible commuters remained unaffected, while for flexible commuters, the unavailability conditions meant a reduction by 9 observations to 2,083.

This approach led to final LL of -1794.86 for the flexible model and -1709.33 for the inflexible model, such that in both cases, the use of constants clearly leads to very important gains in model fit over the basic MNL model, even after taking into account the difference in sample size in the flexible case.

Table 2 summarises the results of this estimation, where only the time period constants are reproduced; the estimates of the other coefficients and constants remained largely unaffected. The results suggest that a single-peak approach would be appropriate in most cases (e.g. outbound and return for flexible commuters), while there are some cases in which multiple peaks should possibly be allowed for, notably along the duration dimension for flexible commuters, where the low significance levels should however be borne in mind. It should also be noted that the results along the outbound dimension for inflexible commuters are inconclusive. Overall, this analysis has shown that significant problems with identification can arise when using a full set of time-period constants. Nevertheless, the analysis has also shown that the approach leads to very significant gains in model performance, showing the importance of acknowledging the time period choice dimension. These findings act as the motivation for our search for a functional form to represent the distribution of the constants.

MNL model with exponential formulation

Given the results from the fully-specified model, it was decided to continue using the 8-17-8 normalisation in the models based on the exponential formulation. This decision was supported by several experiments which showed significant drops in model performance when using alternative specifications, confirming 8-17-8 as the most appropriate single-peak normalisation, in both population segments.

Although not strictly necessary, the extra availability conditions were used again, to enable comparison with the models with constants. The final model fits, along with the estimates of the shape parameters, are summarised in Table 3. Again, the remaining coefficients remained largely unaffected by the treatment of time period choice. In the model for flexible commuters, the use of the exponential formulation leads to an improvement in log likelihood by 124.15 units over the simple MNL model, with 6 additional parameters, which, despite the use of 9 fewer observations (due to the extra availability conditions), is clearly significant. Similarly, in the model for inflexible commuters, the use of the exponential formulation leads to significant gains over the MNL model, by 65.51 units, with 6 additional parameters.

All shape parameters are negative, implying a decreasing slope to either side of the base, and, with a few exceptions, the estimates are all significant at high levels of confidence. An interesting observation can be made for the outbound shape parameters in the inflexible model. Unlike in the flexible model, the right-side parameter is significantly more negative than the left-side one, implying a steeper slope to the right of the base, which can be seen as an effect of the inflexible working hour arrangement, and the accompanying low tendency to accept shifts to a later departure time. A similar observation can be made for the return constants, where the slope is steeper to the right for flexible workers, while it is steeper to the left for inflexible workers. For the duration constants, it is more difficult to find an easy interpretation. To illustrate the effects of the shape parameter, a brief graphical analysis was conducted, with plots shown in Figure 1.

	Final LL	Par.	Adj. ρ^2:	β_{o-}	β_{o+}	β_{r-}	β_{r+}	β_{d-}	β_{d+}
Flexible	***-1855.18***	***22***	0.3006	-0.51 (-6.44)	-0.54 (-5.53)	-0.24 (-2.35)	-0.33 (-5.42)	-0.74 (-3.36)	-0.38 (-4.29)
Inflexible	***-1748.55***	***24***	0.3436	-0.32 (-3.44)	-1.08 (-3.55)	-0.4 (-2.81)	-0.19 (-1.56)	-0.21 (-1.73)	-0.25 (-2.37)

Table 3: Results for models using exponential formulation

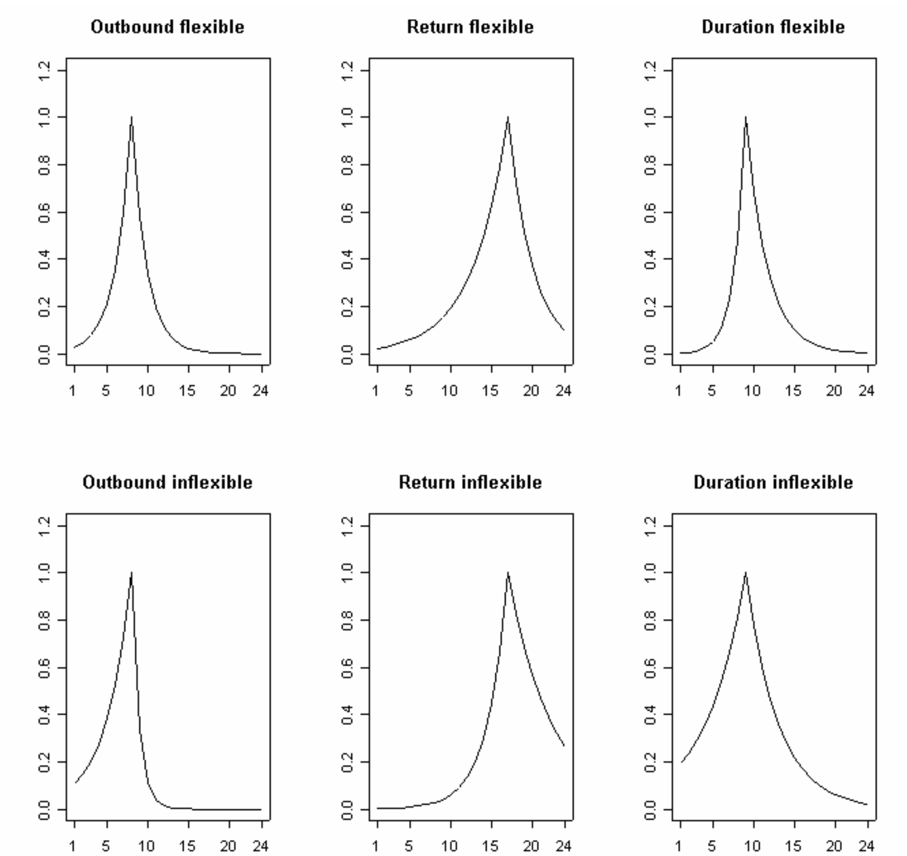

Figure 1: Implied shape of distribution of constants using exponential formulation

The results presented in this section have shown that important gains in model fit can be obtained over the simple MNL model by using a functional form for the constants representing the time period choice dimensions. These improvements were not as important as those obtained when using a full set of constants (124.15 vs 184.47, and 65.51 vs 104.73), but come at a lower cost in terms of the number of additional parameters (6 vs 35, and 6 vs 45). Furthermore, although the additional availability approach was used, this is not strictly necessary with these functional forms.

MNL model with power formulation

Given previous results, the 8-17-8 normalisation was used again in the power formulation. As was the case in the other structures, the results in terms of time and cost coefficients, as well as general dummy variables, remained largely unaffected by the treatment of the time period choice; the results in terms of model fit and shape parameters are summarised in Table 4.
Some problems were encountered with the right-bound duration terms for inflexible commuters, where inconsistent results were already observed with the constants approach. The shape parameter (β_{d+}) took on a value essentially equal to zero (albeit with a significant t-statistic), while the associated exponent (λ_{d+}) took on a very large value. Imposing bounds on these parameters led to unsatisfactory results, such that these values were retained. In terms of model fit, the power formulation, just like the exponential formulation, leads to very significant gains in performance over the simple MNL model, with increases by 153.27 and 69.83 units respectively, at the cost of 12 additional parameters. For flexible commuters, the power formulation leads to gains when compared to the exponential formulation (looking at the adjusted ρ^2 measure), while this is not the case for inflexible commuters. In terms of implied shapes of the distribution, the results still suggest decreasing shapes in the majority of cases, but with notable exceptions, especially for inflexible commuters. This is partly reflected in the findings for the models using a full set of constants.

Flexible commuters						***Inflexible commuters***					
Model fit:	-1826.06					*Model fit:*	-1744.23				
Parameters:	28					*Parameters:*	30				
Adjusted ρ^2:	0.3092					*Adjusted ρ^2:*	0.3430				
	Est.	*T-stat.*		*Est.*	*T-stat.*		*Est.*	*T-stat.*		*Est.*	*T-stat.*
β_{o-}	-0.1108	-1.15	λ_{o-}	2.9033	3.15	β_{o-}	-0.4084	-3.86	λ_{o-}	1.4175	7.00
β_{o+}	-0.4907	-5.90	λ_{o+}	1.1015	9.15	β_{o+}	0.0371	2.36	λ_{o+}	2.0059	13.95
β_{r-}	-0.2718	-2.58	λ_{r-}	1.1784	8.16	β_{r-}	0.0453	0.85	λ_{r-}	1.4976	3.39
β_{r+}	-0.0245	-1.09	λ_{r+}	2.3522	4.24	β_{r+}	-0.4971	-5.34	λ_{r+}	0.9681	8.55
β_{d-}	0.0089	1.45	λ_{d-}	2.6136	9.26	β_{d-}	-0.5211	-4.95	λ_{d-}	0.9202	8.17
β_{d+}	-0.3146	-3.72	λ_{d+}	1.2162	7.05	β_{d+}	10^{-12}	13.92	λ_{d+}	14.5252	121.47

Table 4: Results for models using power formulation

The final step of this part of the analysis consisted of a graphical illustration of the results obtained with the power formulation, as shown in Figure 2, where only the represented part of the ranges of time periods are shown in these figures. The graphs show the increased degree of flexibility of the power formulation when compared to the exponential formulation. They also show that, unlike the exponential formulation, the power formulation is able to pick up the non-strictly increasing slopes to the left of the peak for duration constants in the flexible model, and return constants in the inflexible model, as well as non-strictly decreasing slopes to the right of the peak for outbound and duration constants in the inflexible models. Although these results are less easy to reconcile with the intuitive meaning of the constants than in the case of the exponential formulation, they reflect the results from the model using a full set of constants, and are indicative of the problems of interpretation faced in models calibrated on

SP data. Along with the better fit for this approach, at least in the flexible model, these results show a higher degree of flexibility than was the case with the exponential formulation.

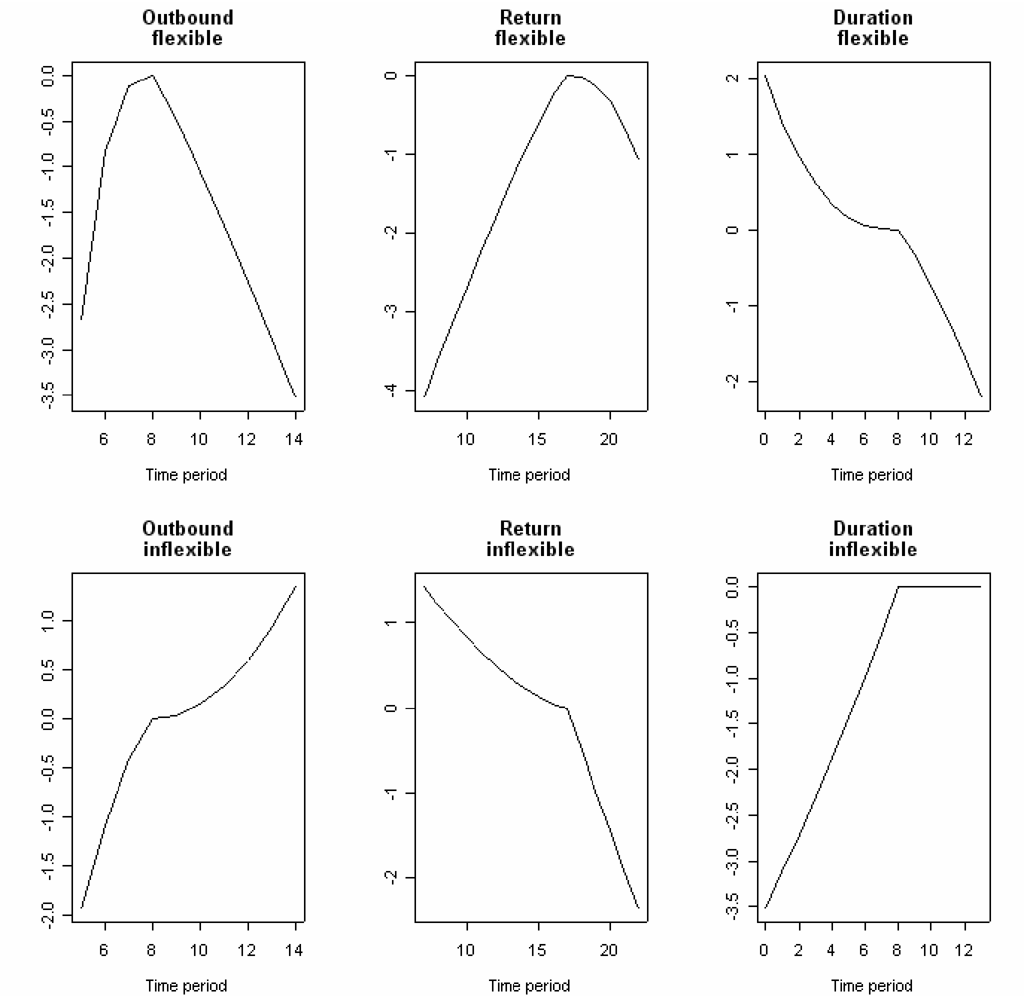

Figure 2: Shape of distribution, power formulation

Empirical distribution

The next step in the analysis consisted of using the empirical distribution approach. For this, an iterative procedure was used. The models were first estimated with the full set of identifiable support points, leading to the equivalent of the model using a full set of constants. Interpolation was then used where appropriate to replace intermediate support points. For ease of analysis, the normalisation β_s=0 was used in all three dimensions. Only the results for the final iteration step are presented here.

For flexible commuters, the base model had a log likelihood of -1794.86, with 51 estimated parameters, which is the equivalent of the model using constants, with availability constraints applied. Progressive iterations led to the replacement by interpolation of outbound constants 6, 12 and 13, return constants 11, 15, and 20, and duration constants 2, 6, 7, 9, 10 and 11. The removal of these 12 intermediary support points led a statistically significant reduction (at the 92% level) in log likelihood by 9.67 units. No further interpolations were possible at this stage without major reductions in model fit. The shape of the distributions in the final model is reproduced in Figure 3. The plots show that in the final model, the three curves are fairly smooth, although the duration curve presented more than one peak. The main exception is the value of return constant 13. The removal of this however leads to a drop in LL by a very significant 6.15 units, which is not acceptable. The estimates of the final model are shown in Table 5.

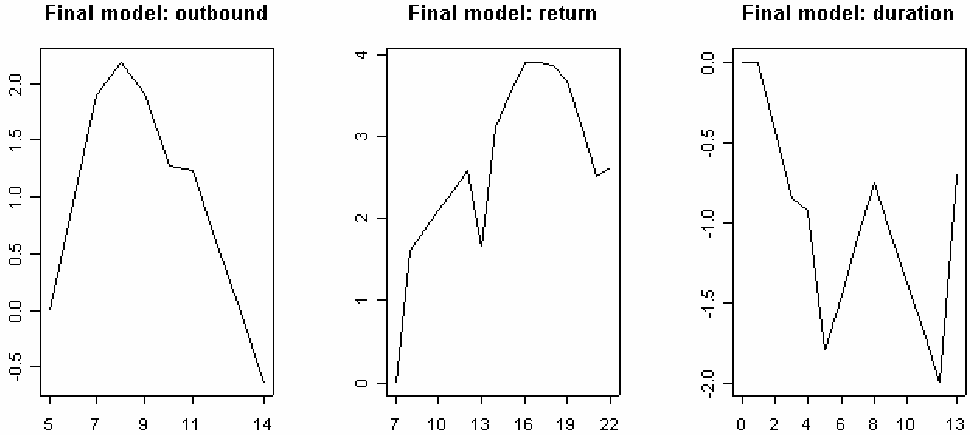

Figure 3: Shape of empirical distribution for flexible commuters

Outbound				Return				Duration			
		Estimate	T-stat			Estimate	T-stat			Estimate	T-stat
Δs = 5	βs	0	-	Δs = 7	βs	0	-	Δs = 0	βs	0	-
Δ1 = 7	β1	1.90	6.03	Δ1 = 8	β1	1.61	1.56	Δ1 = 1	β1	0.00	0.01
Δ2 = 8	β2	2.19	6.10	Δ2 = 12	β2	2.58	2.01	Δ2 = 3	β2	-0.84	-1.37
Δ3 = 9	β3	1.91	4.56	Δ3 = 13	β3	1.65	1.26	Δ3 = 4	β3	-0.93	-1.36
Δ4 = 10	β4	1.27	2.65	Δ4 = 14	β4	3.11	2.38	Δ4 = 5	β4	-1.80	-2.50
Δ5 = 11	β5	1.23	2.25	Δ5 = 16	β5	3.91	2.76	Δ5 = 8	β5	-0.75	-0.89
Δe = 14	βe	-0.64	-0.67	Δ6 = 17	β6	3.90	2.67	Δ6 = 12	β6	-2.00	-1.89
				Δ7 = 18	β7	3.86	2.58	Δe = 13	βe	-0.71	-0.57
				Δ8 = 19	β8	3.67	2.38				
				Δ9 = 21	β9	2.51	1.52				
				Δe = 22	βe	2.63	1.50				

Table 5: Estimates for flexible commuters with empirical formulation

For inflexible commuters, the base model had a LL of -1709.33, with 63 estimated parameters. Progressive iterations led to the replacement by interpolation of outbound constants 6, 7, 9, 11, 12, 18 and 19, return constants 11, 13, 18, 20, 21, and 23, and duration constants 4, 6, 9 and 10. The removal of these 17 intermediary support points led a drop in LL by 4.21 units to -1713.54, where the associated likelihood-ratio test-value of 8.42 has a χ^2_{17} p-value of 0.96, showing that the drop in LL is not statistically significant. The shape of the distributions in the final model is illustrated in Figure 4. The plots reflect the issue of multiple peaks for outbound and return constants, which cannot be accommodated in the exponential and power formulations. The estimates of the final model are shown in Table 6.

The discussion in this section has shown that the empirical approach reduces the number of estimated parameters compared to the full set of constants approach, without major drops in model fit, while avoiding the inflexibility of the parametric approaches. However, some issues of applicability remain, and the potential influence of the SP design on the estimation of the constants is possibly greater than with the parametric approaches. To conclude, it should be noted that the iterative parameter reduction process was conducted manually; the implementation of this approach in estimation software is an important avenue for further research, incidentally also in the investigation of random taste heterogeneity.

Outbound				**Return**				**Duration**			
		Estimate	**T-stat**			**Estimate**	**T-stat**			**Estimate**	**T-stat**
Δs = 4	βs	0	-	Δs = 8	βs	0	-	Δs = 0	βs	0	-
Δ1 = 5	β1	1.66	4.01	Δ1 = 9	β1	-3.10	-2.57	Δ1 = 1	β1	3.13	3.74
Δ2 = 8	β2	3.07	5.81	Δ2 = 12	β2	-1.94	-1.38	Δ2 = 2	β2	2.93	3.16
Δ3 = 10	β3	2.34	3.62	Δ3 = 14	β3	-2.59	-1.74	Δ3 = 3	β3	3.27	3.42
Δ4 = 13	β4	4.10	4.61	Δ4 = 15	β4	-2.10	-1.38	Δ4 = 5	β4	3.07	3.08
Δ5 = 14	β5	4.01	4.19	Δ5 = 16	β5	-2.49	-1.59	Δ5 = 7	β5	4.33	4.00
Δ6 = 15	β6	2.45	1.73	Δ6 = 17	β6	-2.27	-1.42	Δ6 = 8	β6	4.48	4.01
Δ7 = 17	β7	5.12	2.83	Δ7 = 19	β7	-3.11	-1.84	Δ7 = 11	β7	4.35	3.48
Δ8 = 20	β8	7.13	3.91	Δ8 = 22	β8	-4.10	-2.20	Δ8 = 12	β_8	4.03	2.90
Δe = 21	βe	5.99	3.23	Δe = 24	βe	-4.20	-2.12	Δ9 = 13	β9	2.34	1.52
								Δe = 16	βe	6.43	3.51

Table 6: Estimates for inflexible commuters with empirical formulation

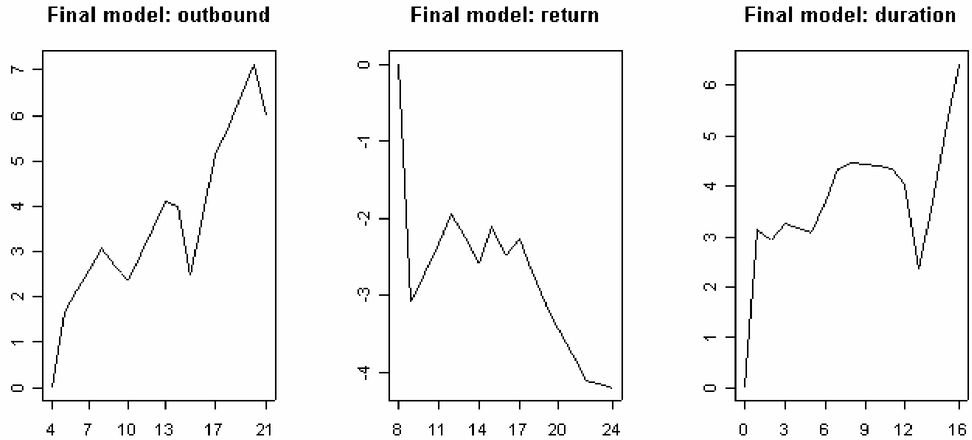

Figure 4: Shape of empirical distribution for inflexible commuters

SYNTHETIC DATA EXTENSION AND MODEL VALIDATION

The analysis conducted in the earlier parts of this paper has highlighted the issues of identification arising with the use of a full set of constants. The analysis has further shown that the use of interpolation between a pair of estimated constants can significantly reduce the number of estimated parameters and increase parameter significance, without major reductions in model fit. This suggests that some simplification of the treatment of constants is possible, and even preferable. The results using the exponential and power formulation support these findings; indeed, even though these approaches lead to significant drops in model fit, these drops are still at an acceptable level, when one considers the important gains in terms of ease of estimation and applicability.

It should be stressed again that the calibration of constants on SP is potentially influenced by a host of factors, notably the design of the SP experiment, and the reaction of respondents to the nature of the survey. This could also partly explain the mixed success with the highly flexible power formulation. As such, the use of RP data is more promising. In the absence of RP data, the decision was taken to generate a synthetic dataset, free of SP errors, for further evaluation of model performance, as well as for model validation.

The construction of the synthetic dataset is based on set of 2,092 observations for commuters with flexible working hours. From this, a sample of 20,920 observations was produced, using small percentage changes on the original attributes for the 2,092 observations (between -20% and +20%). From this sample, 20,000 observations were selected at random to form the estimation sample, with the remaining 920 observations serving as a validation sample. The *true* model was assumed to be one using a full set of constants. However, only the outbound time period dimension was considered in this application, for reasons of simplicity. The aim here is to show the applicability of the functional forms to a *clean* dataset, and the extension to a model using constants along the three dimensions of outbound, return and duration, is straightforward. The values used for the coefficients in the data generation process are those found in the estimation of the model using a full set of constants, where the constants 4, 15, 16 and 17 (for which issues with identification existed) were replaced by linear interpolation.

Five separate models were estimated on this dataset, a simple MNL model, an MNL model using a full set of constants, a model using the empirical distribution approach, a model using the exponential formulation, and a model using the power formulation. The final fits of the different models are shown in Table 7.

Model	**Final LL**	**Parameters**	**Adjusted ρ^2**
MNL	-19625.9	15	0.2344
MNL with constants	-18973.5	28	0.2593
Empirical formulation	-18996.3	24	0.2586
Exponential formulation	-19208.3	17	0.2506
Power formulation	-19029.5	19	0.2575

Table 7: Model fits on synthetic data

The model using the full set of constants was estimated to *validate* the data; the estimates of this model are virtually indistinguishable from the parameter values used in the data generation process, they are all significant at high levels of confidence, and the model has a final LL of -18,973.5 units, with 28 estimated parameters. The remaining four models also perform very well in terms of recovering the 15 parameters not related to the choice of time period (time and cost coefficients, and socio-demographic and SP constants). However, significant differences in model fit exist between the models. As such, the simple MNL model, which does not allow for any treatment of the outbound constants, has a final LL of -19,625.9 equating to a drop in LL by 652.4 units when compared to the model with constants. In the model using the empirical formulation, only 4 parameters could be replaced by interpolation, which nevertheless led to a statistically significant drop by 22.8 units to a final log-likelihood of -18996.3. The model using the exponential formulation yields a final LL of -19,208.3 units, with 17 parameters, thus comfortably outperforming the simple MNL model. Finally, the model using the power formulation obtains a final LL of -19,029.5 with 19 estimated parameters, equation to a drop in LL by a mere 56 units when compared to the model with constants, which although significant, is relatively small, illustrating the flexibility of this approach. The model also clearly gives a better fit than the one using the exponential formulation.

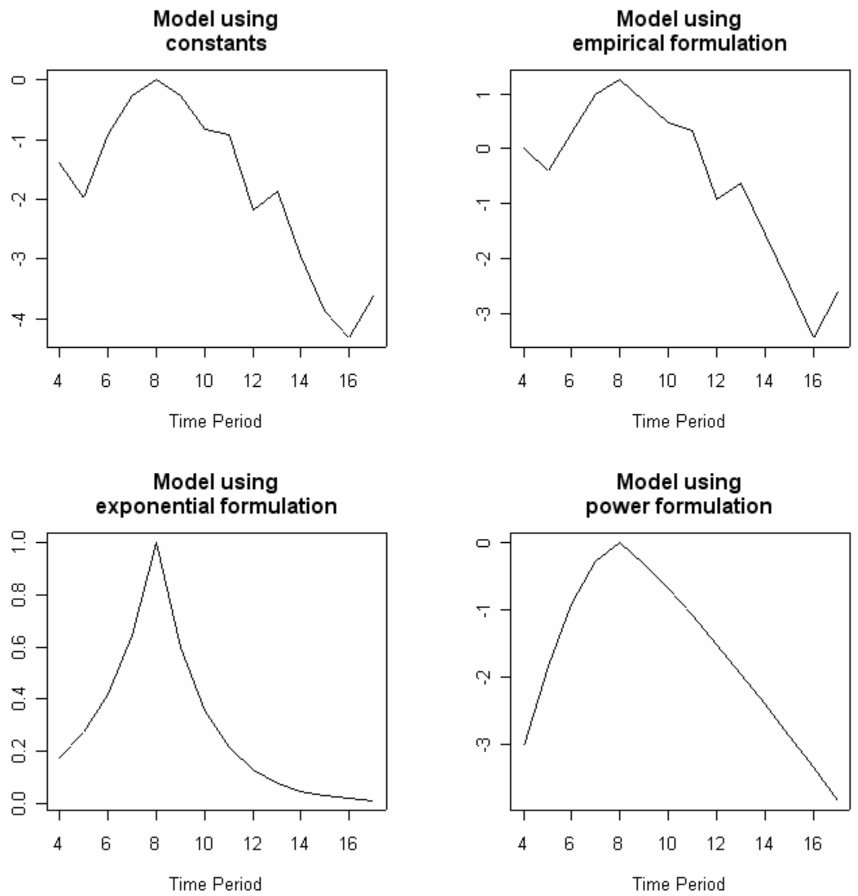

Figure 5: Graphical analysis of models estimated on synthetic data

To conclude this analysis, we conduct a brief graphical experiment (Figure 5), looking at the shape of the implied distribution of the constants in the true model, in the empirical formulation, and in the two functional approximations, using a lower bound at 4, and an upper bound at 17. The results show that the shape assumptions made by the exponential formulation are too strict to allow it to approximate the true shape. On the other side, the power formulation is able to very closely approximate the true shape of the distribution.

As a final step in the analysis, the four models estimated above were applied to the validation sample of 920 observations. The results of the validation exercise are summarised in Table 8, which gives, for each model, the average probability of correct prediction, in addition to the log-likelihood of obtaining the observed choices, and the associated adjusted ρ^2 measure.

As expected on the basis of the relative differences in model fit, the differences in performance on the validation sample are relatively small. Nevertheless, the exercise shows that any model using a treatment of time period choice obtains better performance than the MNL model. As would be expected, the performance on the validation sample is poorer than that obtained on the estimation sample, but within a margin that would suggest that the models have not been overfitted to the estimation data. Although, as mentioned above, the differences in performance are too small to draw any overly reliable conclusions, the results clearly show very good performance by the two parametric approaches (exponential and power), especially when taking into account the lower cost in terms of parameters (as in the adjusted ρ^2 measure). This validation exercise thus suggests that, in the presence of a large number of inter-related constants, these approaches present a viable alternative to the use of a full set of constants. The development of more flexible formulations, notably ones allowing for multiple peaks, is an important avenue for further research.

Model	*Correct prediction*	*Final LL*	*Parameters*	*Adjusted ρ^2*
MNL	42.29%	-949.61	15	0.1878
MNL with constants	43.69%	-933.50	28	0.1904
Empirical formulation	43.67%	-934.32	24	0.1931
Exponential formulation	43.01%	-940.80	17	0.1935
Power formulation	43.53%	-938.43	19	0.1938

Table 8: Prediction performance on validation sample

SUMMARY & CONCLUSIONS

In this paper, we have discussed issues researchers are faced with in the case of models using a high number of constants that have a high level of interdependency. The example used in our analysis is that of time-period choice, where constants are used to capture period-specific unobservables along three dimensions; outbound, return and duration. The potentially high number of constants to be estimated in a fully specified time-period choice model can lead to issues of identification, high computational cost, and problems with interpretation. Due to the heightened correlation between adjacent time-periods, some form of interdependency is clearly to be expected. This, in conjunction with the above discussed problems with a fully-specified model, was the motivation for replacing the full set of constants by a functional approximation. In this paper, we have proposed several alternative approaches, based on parametric as well as non-parametric functional approximations. The analysis has shown that the different approaches all have advantages as well as disadvantages, but that important gains in identification can in general be obtained with the use of a functional approximation.

Although the discussion in this paper has centred on the application to time-period choice-modelling, it should be noted that there are potentially other applications in which the use of a functional form can be preferable to the use of a full set of constants. One example is that of route choice. Another area where the use of a functional form is very promising is that of spatial choice, such as the choice of residential or work location. Here, there are likely to be factors that cannot be captured in full in the observed part of utility, but whose mean varies across space in a clear pattern, for example as a function of the distance to the central unit, the border unit, or the nearest street.

Several avenues for further research can be identified. In terms of the application presented in this paper, these relate to the extension to models allowing for an explicit modelling of the correlation between adjacent time periods, and the representation of random taste heterogeneity across agents. Another important area for future work is the development of parametric functional forms that allow for multiple peaks, and approaches that enable researchers to estimate the location of the peak, rather than fix it a priori. Finally, it is of interest to allow for differences in the shape of the distribution of constants across decision-makers, with one possibility being the use of a random distribution of the shape parameters across agents in the exponential and power formulations.

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